Applied Thermal Engineering 98 (2016) 203–212

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

Analytical analysis and experimental verification of trapezoidal fin for assessment of heat sink performance and material saving

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HIGHLIGHTS

- Analytical and experimental assessment of the performance and material saving of rectangular and trapezoidal fin.
- A new novel dimensionless parameter, r_t representing the ratio of the fin tip to base thickness, is introduced.
- A dimensionless differential equation and close form solutions of temperature and fin efficiency is derived.
- Performance drop depends on r_t ratios, and the most performance difference occurs in $M = 1.7$ to 2.5.
- The analytical results are in line with the experimental results.

ARTICLE INFO

Article history:
Received 23 September 2015
Accepted 21 November 2015
Available online 23 December 2015

Keywords:
Heat conduction
Heat convection
Analytical solution
Rectangular fins
Trapezoidal fins

ABSTRACT

A quick assessment of the performance loss in association with material saving amid rectangular and trapezoidal fin is analytically investigated and experimentally verified. A novel dimensionless parameter – r_t representing the ratio of the fin tip to base thickness – is used to simplify the fin equation. A dimensionless differential equation is derived based on the formulation and the corresponding closed form solution of the dimensionless temperature and fin efficiency is presented. It is found that the performance drop depends on various r_t ratios, and the most pronounced performance difference occurs in the region between $M = 1.7$ and 2.5. Yet the performance difference between the rectangular and trapezoidal fin are quite insignificant at low velocity region; with the increasing of velocity, the performance loss ratio is gradually discriminated to reach the maximum performance loss between region of $M = 1$ and 2, then it begins to converge again toward $M = 4$. The recovering of the ratio is actually associated with the diverging and converging trend of the fin efficiency. The analytical results are in line with the experimental results. At a frontal velocity of 4 m/s, the predicted difference of heat transfer rate between trapezoidal and rectangular fin profile in terms of total heat transfer ratio is only 0.88%.

1. Introduction

In electronic cooling design, air-cooling is still the most widely adapted methods for being reliable and easy to implement. However, due to the poor heat transfer characteristics of air as compared to water, a considerably more fin surfaces are needed for effective heat removal. As a result, the added surfaces may occupy the limited space of the motherboard and often constitutes the major bill of material cost of a cooling solution. Hence it is imperative to design a solution in appropriate size and volume with the least budget. For the past decade, many researchers had devoted efforts to fin design optimization by varying fin geometry. Kraus et al. [1] had addressed important design guidelines for cooling electric devices. Chen and Wang [2] had presented a study related to a novel step or trapezoid surface design applicable to air-cooled heat sink under cross flow condition, the resultant thermal resistance of the proposed trapezoid design reveals a 10% lower thermal resistance than that of the plate fin surface at a specified pumping power. Studies of extended surfaces of longitudinal fins of rectangular, triangular, trapezoidal, concave parabolic, and convex parabolic profiles, and the like were well documented in the monography by Kraus et al. [3]. Mokheimer [4] studied the effect of temperature dependent heat transfer coefficient h on the fin efficiency of annular (radial) fins having rectangular, triangular, concave parabolic and convex parabolic profiles. Dul'kin and Garas'ko [5] developed closed-form inverse solutions (fin parameter as a function of fin temperature) applicable for a straight fin of constant cross-section area subject to a power law type temperature dependent h. A generalized analytical solution that took into account the effect of variable heat transfer coefficient, variable cross-sectional area, and curvature of

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http://dx.doi.org/10.1016/j.applthermaleng.2015.11.131
1359-4311/

the lateral surface of an extended surface was presented by Agwu Nnanna et al. [6]. Aziz and Beers-Green [7] provided information about the temperature distribution, heat transfer rate, fin efficiency, and optimum dimensions of a rectangular fin subject to examine various effects such as convective fin base, wall conduction resistance, and contact resistance between the wall and the fin base. They concluded that contact resistance is essential in designing heat sink.

The earlier work of Aziz and Enamul-Huq [8] used a regular perturbation method to develop an analytical solution for a straight fin with a temperature dependent thermal conductivity; Arslanturk [9] applied the domain decomposition method (ADM) to the similar problem, yielding a six term series solutions for the temperature distribution and the fin efficiency. Rajabi [10] used the homotopy perturbation method to solve the same problem. Kim and Moon [11] showed that a stepped circular pin array of optimum dimensions may be superior in thermal performance to the uniform diameter pin fin array.

Though liquid cooling may be the only candidate for very high flux electronic cooling applications, air-cooled designs are still the main stream cooling technology. This can be made clear from the advent of tablet and phablet first introduced by Apple computer, the CPU chips adopts the ARM architecture that consumes only 1/15–1/20 power of the conventional chips through which can further lengthen the utilization of air-cooling. On the other hand, lowering the volume and weight of the air-cooled heat sink is becoming quite important. This is not just from the size concerns but also from the cost concerns. In essence, cost reduction on every single component especially for the heat sink becomes the imperative issue that leads to the reassessment of a compromised fin profile – trapezoidal fin, which can be designed to contain less weight yet comparable performance to the traditional rectangular fin profile. Related studies of extended surfaces of longitudinal fins of rectangular, triangular, trapezoidal, concave parabolic, and convex parabolic profiles are well documented from Ref. 3. The use of variable area profile fins provide a more efficient utilization of the material, but are usually more expensive to manufacture. Trapezoidal fin profile is considered to be a good choice for replacement of traditional rectangular fin as far as cost reduction is concerned. However, based on the derivation of Kraus et al. [3], the differential equation of the trapezoidal fin shown in Eq. (1) is extremely difficult to transform into a dimensionless form for further analysis.

$$\frac{d^2\theta}{dx^2}+\frac{2\tan(\kappa)}{\delta_a(1-\tan(\kappa))+2x\tan(\kappa)}\frac{d\theta}{dx} -\frac{2h}{k\cos(\kappa)(\delta_a(1-\tan(\kappa))+2x\tan(\kappa))}\theta=0 \quad (1)$$

Aziz and Fang [12] also provided the analytical solutions for rectangular, trapezoidal, and concave parabolic (finite tip thickness). However, this fin analysis is based on two-phase applications; and the differential equation for trapezoidal fin profile is indeed the same as triangular fin profile equation via assuming trapezoidal section as part of the triangular fin profile. In this compromised approach, the manipulation of equation becomes rather complicated and confusing for the initial position ($x = 0$) locating outside the real fin length. Torabi et al. [13] proposed a differential transform method (DTM) to solve the rectangular fin profile and trapezoidal fin profile. However, the solution is lengthy and in the form of combination of finite summation series, and difficult to analyze in dimensionless form. Mosayebidorcheh et al. [14] had adapted least square method (LSM) to solve the equations of rectangular, convex, triangular and concave profiles by the fourth order Runge–Kutta method; yet the trapezoidal fin profile has not been studied in this paper. In real applications for electronic cooling industry, trapezoidal fin profile is the only choice with lower cost and easy manufacturing process compared to rectangular fin profile.

Most studies cited in the preceding sections, either by the approach of numerical solution LSM method [14], DTM method [13], ADM method [9], or using the triangular profile fin equation with different boundaries to obtain the solution indirectly [12]. In the present study, we analyze the fin performance by a second-order differential equation which is solved by applying two boundary conditions, one at the fin base and the other at the fin tip with adiabatic or convective boundary condition and solutions are well depicted by Incropera et al. [15]. Normally the base is set as constant temperature or constant heat flux, while the tip is normally set as adiabatic for simplification. In electronic cooling industry, the fin thickness is very thin and heat is barely dissipated from the tip as compared with the whole fin surface. Hence, adiabatic tip assumption holds with an acceptable accuracy and can be derived more easily. The foregoing studies, however, did not provide a quick evaluation of how the taper angle affects the weight loss and performance loss between the rectangular fin profile and trapezoidal fin profile. In real applications, thermal solutions are always slightly over designed to meet the worst case scenario; and clients often accept a small performance drop in trade of 10–20% reduction as long as thermal modules are 100% screened from the production lines.

As a consequence, a quick assessment of the performance and cost issue is essential for the design engineers. Fast evaluations of the appropriate taper angle in association with the performance drop and cost reduction is essential to meet the demand of the real world. It is therefore the objective of this study to propose a novel dimensionless parameter – r_t which represents the tip to base thickness ratio, and it can be used to simplify the equation from Kraus et al. [3] Note that the rectangular fin was examined by Kraus et al. [3] with a constant cross-sectional area and was solved by imposing a constant temperature on fin base, and adiabatic condition at fin tip for boundary conditions. Results for rectangular profile and trapezoidal profile in both cases are discussed. The criterion, under which the fin can have a prescribed temperature and the fin tip is adiabatic with zero heat flux flowing outward. In this study, by introducing the r_t and fin length b, the taper angle for trapezoidal fin profile then can be defined as Eq. (2).

$$angle=\tan^{-1}\frac{t_b(1-r_t)}{2b} \quad (2)$$

The purpose of this paper is to present a quick assessment of the relationships between weight loss (or cost saving) against the performance loss. The results can assist thermal engineers with a much more direct sense about what performance loss in exchange of the cost reduction of material without going through tedious calculation processes.

2. Mathematical models

2.1. Mathematical analysis

As seen from Fig. 1, the rectangular fin has a length b and thickness t while the coordinate x is measured from the fin tip (denoting $x = 0$). However, the traditional geometry terminology set the trapezoidal fin at the fin tip location at $x = x_e$, yet the position $x = 0$ is outside the real fin length. As a result, the length of rectangular fin is b; yet the length of trapezoidal fins is $b - x_e$, and it is unable to quantitatively compare the fin performance on equal basis. To tackle this problem, the present study proposes a new parameter r_t, to have a length b and thickness t which is the same as rectangular fin. Note that the coordinate x is measured from the fin tip on $x = 0$; the trapezoidal fin equation is re-derived by this parameter rather than from