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Research Paper

Heat transfer from warm water to a moving foot in a footbath

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HIGHLIGHTS

- Increasing perfusion rate or the skin resistance leads to reduced heat in the fat layer.
- The foot in motion greatly alters the linear trend of the temperature in the fat layer.
- Upward motion of the foot heats, but downward action cools both the fat and skin layers.
- Less time is required to approach a steady stage when the foot moves downwards.
- In terms of gaining better thermal sensation, downward movement of the foot is desirable.

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ABSTRACT

A model is proposed in this work to investigate the impact of a moving foot on the transfer of heat from a constantly heated warm water into the foot immersed within a footbath. The model is simply based on the well-known Pennes bioheat equation that governs the thermal process of the fat part of the foot in action. Both the steady and transient temperature responses of the foot are analytically evaluated. The visible physical effects of the up and down movement of the foot on the skin layer heat and also on the scaled heat flux in the fat layer are later explored and discussed. It is found that the upward motion heats up the foot whereas the downward motion cools down the foot leading to a better thermal sensation and comfort during the thermal process of the foot in the footbath which is highly desirable during a physiotherapy.

1. Introduction

Since the Pennes bioheat equation [1,2] has been successfully employed for the heat transfer processes in the past in living tissues, such as in the physiological [3] and medical studies [4], the present work focuses on the thermal analysis of a moving foot immersed in a footbath via the Pennes bioheat model in an aim to explore foot movement effects on the temperature distribution through the foot layers.

The Pennes bioheat model, when compared to its variations, is simpler to analyse and to explain the phenomena in several thermal processes in real life situations. For instance, the phenomenon within a blood vessel was modelled in Reference 5. The temperature distribution within a human eye subjected to a laser source was studied by the model in Reference 6. More laser–tissue interaction applications may be found in Reference 7. The temperature response of a semi-infinite biological tissue due to a sinusoidal heat flux at the skin was solved in Reference 8. The brain hypothermia was also explained by the Pennes bioheat model together with an experimental validation in Reference 9. Further advances and mechanical applications may be found in References 10 and 11. Steady solutions for one-dimensional bioheat transfer model of the living tissues were derived in Reference 12. Temperature elevation generated by ultrasonic irradiation in biological tissues was estimated in Reference 13. The catheter ablation therapy, with particular application to RF thermal ablation on the atrial tissue, was studied in Reference 14. Temperature in live biological tissues of different organs was calculated in Reference 15. Theoretical and numerical studies regarding the fractional Pennes bioheat equation were given in Reference 16. It is noted that the analytical solutions in the above cited references are too specific for the considered problems.

Warm footbath is generally used as a physiotherapy tool to get a healthy body by increasing the feet temperature above the normal. It was shown that the bathing of a foot in a warm water can lead to better circulation in blood [17] and it considerably increases the quality of sleep [18]. The Pennes bioheat model was recently used to compute the temperature distribution in the structured foot residing in a footbath [19]. The thermal sensation and comfort observed in an accompanying experiment conducted in this article were well explained by means of the analytical results presented from the

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Pennes bioheat equation. To briefly expand, in Reference 19 a layered structure of the foot was adopted, such that the skin layer was represented by a boundary constraint with a thermal resistance through the skin and a fixed temperature boundary condition was used for the core layer. These simplifications enabled the formulation of the physical phenomenon as a one-dimensional heat conduction problem through the Pennes bioheat model. A well-controlled environment chamber was then employed for the purpose of experimental evidence and a questionnaire survey was made to record the subjective thermal sensation. It was eventually concluded that the heat flux through the skin layer is capable of describing the participants' subjective thermal sensation, which indicates a possible close relation between subjective thermal sensation of footbath and objective temperature responses.

The present motivation, therefore, is to extend this model to incorporate the effects of the foot when it is not resting as in the past literature studies, but in an up or down motion. It is targeted to investigate how such a mechanism of the foot can contribute to the heat transfer from the warm water into the foot so that possible relaxing and comfortable properties of the footbath during a thermal treatment can be predicted and understood. Indeed, the present mathematical analysis suggests that foot movement in a footbath can be more relaxing than a static immersion, which could be used to relax human feet during an individual footbath as already implemented in some Asian countries and regions [19] or possibly to cure at a thermal spring, thermotherapy. This scientific finding, of course, must be supported and validated by experimental data that remain open.

2. Mathematical modeling of the temperature field

In parallel to the theoretical and experimental study in Reference 19, a skin, a fat and a core zone constitute a three-layer structure of a foot revealed in Fig. 1a–c. We primarily consider the thermal process of heat transferred along the thickness x from warm water with a uniform temperature T_w through the skin and fat layers into

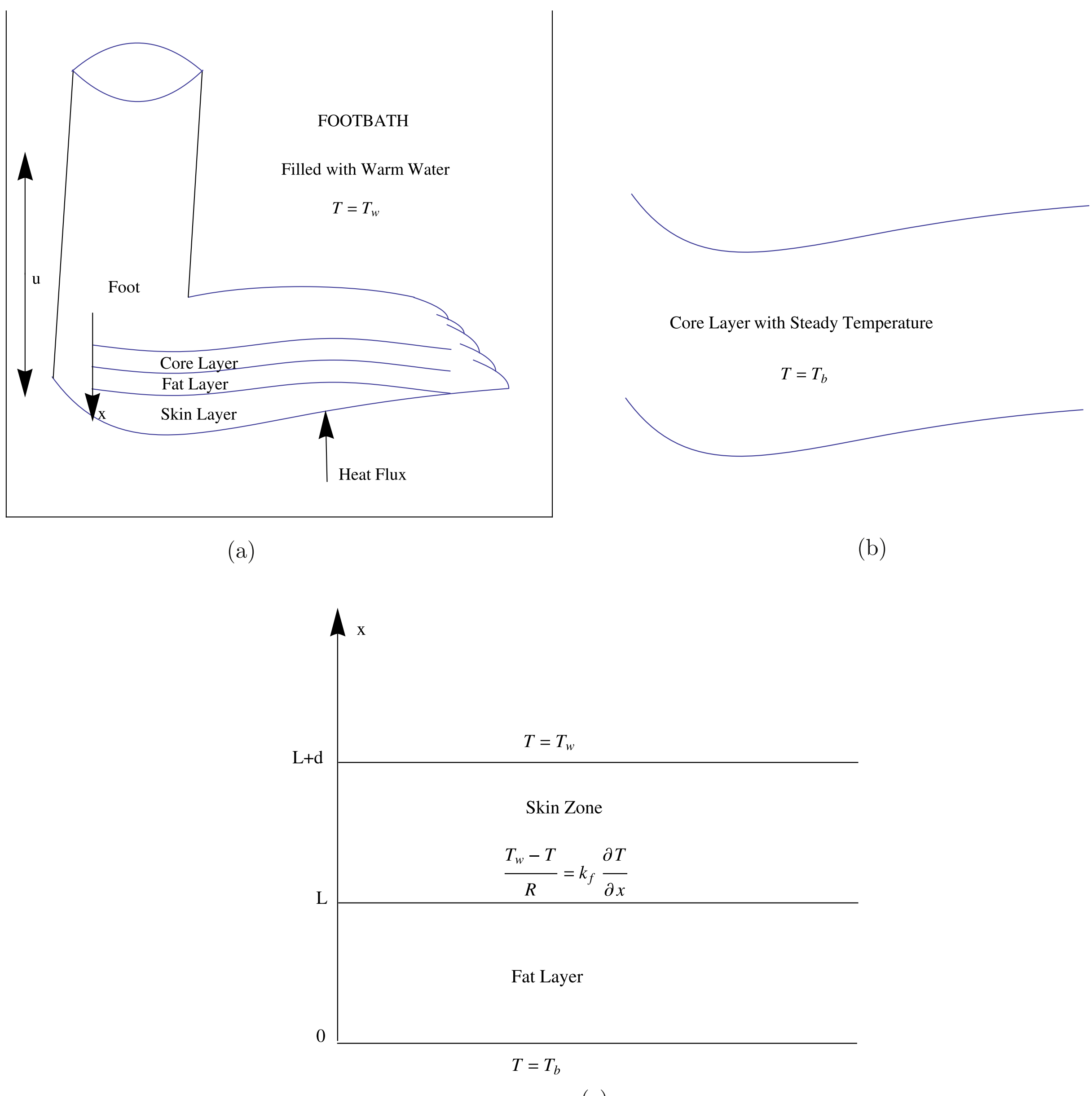

Fig. 1. Basic layout of the physical model.