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A state space synchronous machine model with multifunctional characterization of saturation using Levenberg–Marquardt optimization algorithm

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ABSTRACT

Performance of electrical machines is critically affected by the saturating nature of ferromagnetic materials. In order to have a reliable and accurate estimation of the steady-state behavior of synchronous machines, it is important to have a precise machine model that includes saturation. This paper aims to develop a new synchronous machine model incorporating an accurate saturation model. The algorithm used in this paper to represent magnetic saturation calculates the coefficients in several real function expressions for the saturation characteristics of a synchronous machine. The new synchronous machine model proposed in this paper has been used in the investigations to calculate the machine behavior in various operating conditions.

1. Introduction

To ensure an accurate and reliable model for electrical machines, it is necessary to use a precise and accurate mathematical representation of the magnetizing saturation. The magnetic flux in the direct and quadrature axes of synchronous machines is influenced by the saturation phenomenon. Thus, it will be useful to have a synchronous machine model integrated with an accurate universal saturation model in algebraic configuration that makes it valuable in understanding the system behavior. Typical saturation characteristics of an electrical machine is presented in Fig. 1. At low magnetizing current values, the flux linkage is proportionately related to the current. This region is called the unsaturated region. For high values of the magnetizing current, the flux linkage in the machine reaches its maximum level, which is known as the highly saturated region. Transition between unsaturated and highly saturated regions takes place in the non-linear region. As illustrated in Fig. 1, in this region the flux linkage is not proportionally related to the magnetizing current [1–6].

The concept of steady-state stability in electrical machines implies that small changes in the terminal inputs, the initial conditions, or the machine parameters do not result in severe changes in the machine output. Similar to all other working systems, synchronous generators are designed to work in a stable situation. Therefore, the study on stability boundaries of synchronous machines results in performance improvement [7,8]. To make the results more reliable, it is essential to consider the saturation effect in the machine model. Various techniques such as polynomial, rational-fraction, sinusoidal, and trigonometric functions are intended to address inclusion of the saturation phenomenon into the electrical machine model [9–16]. In [17], a saturation model is proposed to be used in the steady analysis of synchronous machines. The saturation model is obtained as a result of correcting the unsaturated *d*- and *q*-axis reactances by multiplication factors. A synchronous machine model considering magnetic saturation and hysteresis effect is proposed in [18]. The machine, under study, has been connected to an unbalanced load, and the transient and steady-state analysis have been carried out accordingly. Authors in [19] have developed a synchronous machine model and investigated the saturation effect on the machine response following a five-percent reduction in the machine terminal voltage. Results show that, although there is a considerable discrepancy between the saturated and unsaturated results, the mutual inductance between the *d*- and *q*-axis can be neglected. This results in a trade-off to have a simplified model against acceptable errors in the results. In [20], an optimization algorithm, based on the Levenberg–Marquardt (LM) method, is developed to represent the flux linkage and magnetizing current relationship in permanent magnet synchronous generators. As illustrated in Fig. 1, any combination of linear or nonlinear functions can be chosen by the designer to represent the saturation characteristics.

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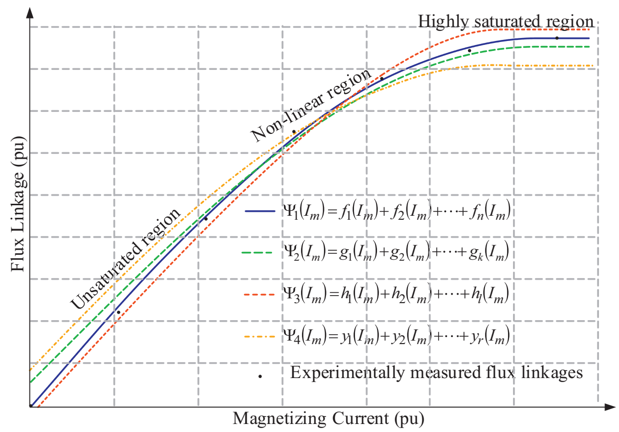

Fig. 1. Different configurations to represent saturation characteristics of a typical electrical machine.

The objective of this paper is to develop a new state space model for synchronous machines integrated with the saturation model in [20]. The algorithm, presented in Section 2 of this paper, has been improved and modified to represent synchronous machine saturation with several configurations in a specific domain of applicability. Applying the saturation algorithm, the coefficients can be defined to such an extent to find the most accurate description for saturation characteristics. This has been verified in Section 3, by comparing the results on different saturation configurations provided by the LM method and some other existing methods. In Section 4, the proposed state space model for synchronous machines with the inclusion of magnetic saturation is presented. This has led to a comprehensive and complete model, which can be used for performance analysis of synchronous generators. In Section 5, it has been demonstrated that the proposed algorithm provides an accurate synthesis for the steady-state analysis of synchronous machines.

2. Multifunctional regression algorithm to represent the saturation characteristics

The main objective of this section is to explain the LM algorithm that can be used to represent the saturation characteristics of synchronous machines in accordance with any configuration selected by the designers. This model will then be compared to the existing saturation models, such as polynomial, rational-fraction, DFT, and trigonometric methods. The calculated results presented in Section 3 show that this model can be used as an alternative for aforementioned methods.

2.1. The discrete Fourier transform and the trigonometric curve fitting methods to represent saturation

The main purpose of the DFT and the trigonometric curve fitting methods is to represent the saturation characteristics curve (Ψ–I_m) in a sinusoidal format, based on a set of n measured data points such as $[(I_{m1}, \Psi_1), (I_{m2}, \Psi_2), \ldots, (I_{mn}, \Psi_n)]$.

In [13], the DFT of the mirrored signal can be expressed as in (1).

$$\Psi(I_m) = b_0 + \sum_{j=1}^{k'} b_j \cos(\omega_j I_m) \tag{1}$$

in which the coefficients can be defined by (2) and (3),

$$b_0 = \frac{1}{I_{\max}} \int_0^{I_{\max}} \Psi(i)di = \frac{1}{I_{\max}} \sum_{k=2}^{n} \times \left\{ \Psi_k(I_k - I_{k-1}) - \frac{1}{2}\gamma_k (I_k - I_{k-1})^2 \right\} \tag{2}$$

and

$$b_j = \frac{2}{I_{\max}} \int_0^{I_{\max}} \Psi(i)\cos(\omega_j i)di = \frac{1}{I_{\max}} \sum_{k=2}^{n} \times \left[\frac{\gamma_k}{\omega_j}(I_k - I_{k-1})\sin(\omega_j I_{k-1}) + \frac{\gamma_k}{\omega_j^2}\{\cos(\omega_j I_k) - \cos(\omega_j I_{k-1})\} + \frac{\Psi_k}{\omega_j}\{\sin(\omega_j I_k) - \sin(\omega_j I_{k-1})\} \right] \tag{3}$$

where γ_k, ω_j, and $I_{\max}$ are defined as in (4)

$$\left.\begin{aligned} I_{\max} &= I_{mn} \\ \omega_j &= \frac{j\pi}{I_{\max}} \\ \gamma_k &= \frac{\Psi_k - \Psi_{k-1}}{I_k - I_{k-1}} \end{aligned}\right\}. \tag{4}$$

Despite the fact that this technique is simple to implement, its accuracy is highly influenced by the number of sinusoidal terms. In order to obtain more accurate results, the DFT saturation model should include a greater number of sinusoidal terms.

The trigonometric method to represent saturation characteristics can be performed through the determination of the frequency and amplitude of each sinusoidal term. The developed trigonometric function is expressed as:

$$\Psi(I_m) = \sum_{i=1}^{k'} [\alpha_i \cos(\upsilon_i I_m) + \beta_i \sin(\upsilon_i I_m)] \tag{5}$$

where k' $(2k' < n)$ is called the order of the trigonometric series, $(\alpha_i)_{i\in[1:k']}$ and $(\beta_i)_{i\in[1:k']}$ are the amplitudes and $(\upsilon_i)_{i\in[1:k']}$ is the frequency of each trigonometric term[16].

2.2. Non-linear optimization algorithm to represent magnetic saturation

In this paper, a non-linear optimization method, namely, the Levenberg–Marquardt (LM) algorithm [20–22], is used to represent the machine saturation characteristics for a set of given experimental data points. The advantage of applying this technique is that the saturation characteristics of the synchronous machine can be represented by a series of non-linear multi-variable functions. It should be noted that the coefficients might also have different dimensions.

Suppose a set of experimentally obtained magnetization data points is expressed as

$$\left.\begin{aligned} \hat{\boldsymbol{\Psi}} &= [\hat{\Psi}_1 \quad \hat{\Psi}_2 \cdots \hat{\Psi}_n]^T \\ \hat{\mathrm{I}} &= [\hat{I}_{m1} \quad \hat{I}_{m2} \cdots \hat{I}_{mn}]^T \end{aligned}\right\} \tag{6}$$

where $\hat{\mathrm{I}}$ is the measured magnetizing current data point matrix and $\hat{\boldsymbol{\Psi}}$ is its corresponding measured flux data point matrix. To fit the function $\Psi(I_m)$, to the data points in (6), the LM algorithm starts by using the chi-square error criteria to minimize the sum of the