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An efficient algebraic approach to observability analysis in state estimation

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ABSTRACT

An efficient and compact algebraic approach to state estimation observability is proposed. It is based on transferring rows to columns and vice versa in the Jacobian measurement matrix. The proposed methodology provides a unified approach to observability checking, critical measurement identification, determination of observable islands, and selection of pseudo-measurements to restore observability. Additionally, the observability information obtained from a given set of measurements can provide directly the observability obtained from any subset of measurements of the given set. Several examples are used to illustrate the capabilities of the proposed methodology, and results from a large case study are presented to demonstrate the appropriate computational behavior of the proposed algorithms. Finally, some conclusions are drawn.

1. Introduction

1.1. Motivation and approach

By observability analysis one refers to the problem of identifying if a set of available measurements is sufficient to estimate the state of an electric energy system or of a part of it. Observability analysis deals not only with the number of measurements but also with their types and locations. If the system state is observable, identification of critical measurements, understanding as such those that, if missing, render the state unobservable, is crucial.

Contrary, if the state of the system is unobservable, it is relevant to identify observable islands, i.e., those areas of the system whose respective states can be estimated, and to identify which minimum set of additional measurements render the whole system observable.

We provide a simple algebraic algorithm involving low computational burden that at the same time presents good updating characteristics. It is based on transferring rows to columns and vice versa of the relationships among measurements and state variables in the Jacobian measurement matrix.

1.2. Literature review

Observability techniques for electric energy systems can be classified as topological [1–4], which generally involve combinatorial computational complexity, algebraic [5–14], and hybrid [15–17].

The authors of [1] present the theoretical bases of a topological algorithm to study the observability of the state of a power system. The proposed combinatoric algorithm is based on building a spanning tree of full rank contained in the network. In [2], the theory developed in [1] is extended to obtain the largest observable subnetworks of an unobservable power system. In [3], the authors present a topological algorithm which searches for an observable spanning tree using an algorithm based on matroid intersections. A topological algorithm based on the augmenting-sequence concept to build a maximal forest of full rank is presented in [4], where islands are merged into a single node to improve the computational behavior.

As an alternative to the graph-based topological algorithms, in [5] an algebraic technique based on the triangular factorization of the Gain matrix is developed. According to that technique, two iterative algorithms for islands identification and measurement placement are proposed in [6]. In [7,8] the proposed algebraic methodology is also based on factorizing the Gain matrix, but the authors introduce direct methods for islands identification and multiple measurement placement. In [9], the proposed technique is based on the Jacobian matrix instead of the Gain matrix. The Peters-Wilkinson decomposition of the Jacobian matrix permits analyzing the observability of the state of the system and classifying the measurements as critical and redundant. In [10], the author proposes to extract all the information for observability purposes from the Jacobian measurement matrix using Gaussian elimination techniques. A procedure relying on the Jacobian measurement matrix is proposed in [11–13], where the authors present a methodology based on calculating the null space using an elaborated pivoting strategy. Ref. [12] provides a unified approach to observability based on an orthogonal pivoting strategy to obtain the null space of a matrix,

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and then explains how this method can be used to identify redundant and critical measurements, irrelevant boundary injections, pseudo-measurements required to attain observability, replacements that preserve observability, observable islands, etc. In [14] an algebraic algorithm based on the Gram matrix is introduced.

In [15–17] a hybrid (algebraic-topological) method is proposed to study the observability of the state of a power system. In [15], the available flow measurements are associated with branches forming islands that determine a reduced network. Algebraic techniques are then applied to this reduced network for observability purposes. In [16,17], a hybrid formulation based on network graph properties and the reduced echelon form of a test matrix is developed.

Appropriate observability (and state estimation) background is provided in [18,19].

1.3. Problems addressed and approach

Consider the measurement vector equation:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e}, \tag{1}$$

where $\mathbf{h}(\cdot)$ is the m-dimensional nonlinear measurement function, $\mathbf{x}$ is the n-dimensional state vector, $\mathbf{z}$ is the m-dimensional constant vector of measurements and $\mathbf{e}$ is the measurement-error vector.

For observability purposes [18], Eq. (1) can be linearized and errors neglected:

$$\Delta\mathbf{z} = \mathbf{H}\Delta\mathbf{x}, \tag{2}$$

where $\mathbf{H}$ is the $m \times n$ Jacobian measurement matrix of the available measurements. We denote by $\bar{\mathbf{H}}$ the $r \times n$ Jacobian measurement matrix of all candidate measurements not included in $\mathbf{H}$. A candidate measurement is a non-available measurement that can be incorporated into the measurement system at a certain cost. If these measurements are used to identify observable islands, then, at least one flow measurement per branch must be available or candidate. Note that the candidate measurements are both pseudo-measurements to restore the observability of the system and planned measurement installations designed for future expansion of the measurement system.

Initially, we start from the linear relationship provided by the Jacobian matrix $\mathbf{W} = \begin{bmatrix} \mathbf{H} \\ \bar{\mathbf{H}} \end{bmatrix}$, where all the state variables correspond to the columns of matrix $\mathbf{W}$, i.e., they are non-observed, and all the available and candidate measurements correspond to the rows of matrix $\mathbf{W}$.

At each step of the proposed procedure, some state variables and measurements are linearly related with others as provided by $\mathbf{W}$. In this paper, different types of measurements and state variables are differentiated as a function of their properties and *position* in the Jacobian matrix (row or column), as follows:

(1) Available measurements: measurements available for observability purposes.
(2) Candidate measurements: measurements not available, but accessible at a certain cost.
(3) Essential measurements: available measurements which are transferred to the columns of matrix $\mathbf{W}$.
(4) Redundant measurements: measurements related to the rows of matrix $\mathbf{W}$ that depend only on the essential measurements. They can be both available or candidate.
(5) Non-redundant measurements: measurements related to the rows of matrix $\mathbf{W}$ that depend not only on the essential ones, but also on some state variables. They can be both available or candidate.
(6) Critical measurements: essential measurements that, if missing, render the state unobservable.
(7) Irrelevant boundary injection measurements: injection measurements at buses with unobservable branches.
(8) Observable state variables: state variables related to the rows of matrix $\mathbf{W}$ that depend only on the essential measurements.
(9) Non-observable state variables: The state variables related to the columns of matrix $\mathbf{W}$. Also, we denominate non-observable state variables those related to the rows of matrix $\mathbf{W}$ that depend not only on the essential measurements but also on some non-observable (column) state variables.

This paper provides a robust algorithm that simultaneously allows us checking observability, identifying critical measurements and irrelevant boundary injections, determining observable islands and selecting measurements to restore observability. The algorithm transforms the Jacobian measurement matrix using a procedure based on Gauss elimination. It progressively expresses state variables as a function of available measurements, i.e., it transfers columns to rows. Alternatively, other authors have proposed to factorize the Jacobian measurement matrix or the gain matrix (see, e.g., [8,9]). Note that the proposed transformation is not a factorization. Since the $\mathbf{W}$ matrix relates some measurements with the essential ones, one converts in essential those measurements which have been transferred to the columns of $\mathbf{W}$ and determines if the values of other measurements and state variables can be calculated in terms of these essential ones. If this is the case, these variables and measurements become observable and redundant, respectively. By the row-column exchange procedure used in the algorithm one tries to use as columns the available measurements. Since one starts with the (non-directly observable) state variables as columns, and they constitute a minimum set of linearly independent variables, replacing one state variable by a measurement with associated non-null coefficient in matrix $\mathbf{W}$ (the pivot), guarantees that the resulting system of equations is equivalent to the previous one, as shown in Section 3.2. If all state variables can be expressed in terms of available measurements, the system state is observable, otherwise, it is not.

1.4. Contributions

The contributions of this paper are as follows:

(1) The proposed algebraic algorithm is based on a transformation to express state variables as a function of measurements, based on well-known and robust pivoting operations.
(2) The observability information obtained from a given set of measurements also supplies the observability information associated with any subset of measurements of the given set.
(3) The output table supplied by the proposed algorithm permits obtaining the observable state variables and the redundant measurements, not only at the end of the process but in each of its steps.
(4) A single procedure allows determining both measurement replacements (observable case) and observable islands (non-observable case).
(5) Observable state variables and redundant measurements are easily identified through the analysis of the resulting matrix. It is sufficient to check for zero elements in the row associated with the variable or measurement under study and the columns of the non-observable state variables.
(6) The output table of the proposed algorithm permits obtaining the critical measurements, the observable branches and the irrelevant boundary injections.
(7) A simple procedure for identifying the observable islands based on the final matrix information is given.