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Research Paper

# Thermally developing flow inside a porous-filled channel in the presence of internal heat generation under local thermal non-equilibrium condition: A perturbation analysis

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## HIGHLIGHTS

- Analytical study of thermally developing forced convection inside porous materials.
- Performing a perturbation analysis to avoid utilizing thermal boundary condition for constant wall heat flux.
- Effects of internal heat generation/absorption on Nusselt number and thermal entry length.
- Presenting the condition of zero-thermal entry length.

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## ABSTRACT

In the present work, a perturbation analysis is performed to study the thermally developing forced convection heat transfer inside a channel filled with a porous material under local thermal non-equilibrium (LTNE) condition. Internal heat generations within the solid and fluid phases are considered. Channel walls are subjected to a constant heat flux. It is assumed that there is a small temperature difference between the fluid and the solid phases of the porous material. So, performing a perturbation analysis enables us to avoid utilizing models for the constant wall heat flux boundary condition to investigate the hydrothermal behavior of the system. Therefore, analytical solutions are developed for temperature difference between the solid and the fluid phases as well as the local Nusselt number in the porous medium. Effects of pertinent parameters such as dimensionless axial length, Biot number, effective thermal conductivity ratio and dimensionless heat generation parameters on the Nusselt number are discussed. To further clarify the validity of the solution provided, the obtained results are compared with the solutions for two primary approaches (Models A and B) for the constant wall heat flux boundary condition. Results show that both the Nusselt number and the thermal entry length increase with the increase of thermal conductivity ratio. The Nusselt number and the thermal entry length are found to decrease with the increase of the internal heat generation of the solid phase. It is further observed that the Nusselt number and the thermal entry length are less sensitive to the solid internal heat generation at high Biot numbers. Finally, it is found that when the effective thermal conductivity ratio tends to infinity, the thermal entry length tends to zero for high Biot numbers.



## 1. Introduction

Many analytical studies have investigated the problem of forced convection heat transfer in porous materials due to its high academic and industrial significance [1,2] such as electronic cooling, heat pipes, nuclear reactors, drying technology and multiphase catalytic reactors. In majority of these applications, a porous material is inserted between the two parallel plates while the walls of the plates are subjected to constant heat flux or constant temperature. In such case, the undeveloped thermal boundary layer leads to a larger heat transfer rate in the thermal entrance region compared to that of the corresponding thermally fully developed region [1]. One of the main characteristics appearing due to the existence of a porous medium is that the local temperature difference between the solid and the fluid phases can substantially influence the heat transfer process [3] and hence a proper analysis of the heat transport through the porous medium is required.

There are two ways of modeling the energy equation in a porous medium: local thermal equilibrium (LTE) and local thermal

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non-equilibrium (LTNE) models [3]. The LTE model holds when the temperature difference between the solid and the fluid phases is negligible. For some applications of porous materials, however, this temperature difference is considerably large. Hence, utilizing LTNE model is required. However, the use of LTNE model under the constant wall heat flux thermal boundary condition requires additional information to account energy transport between the two phases at the channel wall [4,5]. This information is usually provided in the form of models related to the constant wall heat flux boundary conditions [6,7]. This, in turn, makes the thermal behavior of the system dependent upon the applied model. Extra levels of complexity are hence added to the problem, which involve devising the proper models and including them in the analysis. The applicability conditions of two common models of the constant heat flux thermal boundary condition were mentioned by Dehghan [8]. In the present analysis, a perturbation analysis is performed to avoid the use of the extra models for the constant heat flux boundary condition. Most recently, Dehghan et al. [9] have demonstrated that model A [7,8] is the only applicable thermal boundary condition model in the slip-flow regime.

While the forced convective heat transfer is naturally a developing phenomenon (i.e. the temperature profile pattern varies in the thermal entry region as well as the heat transfer rate), only few analytical studies considered the developing section listed in the following. Nield et al. [10] and Kuznetsov et al. [11] used modified Graetz methodology to investigate the thermally developing heat transfer through a porous material sandwiched in a channel or a tube with walls at constant temperature. Nield and Kuznetsov [12] extended the study of References [10,11] to the non-Newtonian fluid flows. Haji-Sheikh et al. [13] found the temperature profile as well as the Nusselt number in the thermal entrance region of rectangular porous passages. Kuznetsov and Nield [14] studied thermally developing forced convection through a porous material in the slip-flow regime. Ouyang et al. [15] utilized three different models for the constant heat flux boundary condition (the so called Models A, B and C) to find the Nusselt number of thermally developing forced convection through porous materials. Dehghan et al. [16] discussed the validity of three models used by Reference [15] in the entrance region. They showed that these models may yield results that cannot be interpreted in terms of physics of the problem.

The role of internal heat generations within both the solid and the fluid phases, on the temperature field within the porous medium, has been given attention recently [7–9,17–19]. The internal heat generation and absorption may arise from different sources, such as magnetic heating, endothermic/exothermic reactions and viscous heating in both the solid and the fluid phases of a porous material. In addition, using porous materials as catalysts are usual in chemical engineering applications from burners to refiners. For example, a clean fuel such as hydrogen can be obtained from endothermic reactions of methane reforming inside a porous catalyst coated with Palladium nanoparticles [20]. Meanwhile, a most recent application of porous materials with heat generation/absorption is artificial porous media obtained by grids of PCMs (Phase-Change Materials). These PCMs can act as endothermic or exothermic media. To analyze the problem analytically, the heat generations/absorptions are assumed constant volume-averaged values to find the overall thermal response of such systems. Understanding the thermal response of such systems is mandatory in designing these systems and in selecting the materials involved. Heat generations/absorptions appear as source/sink terms in the energy conversation equations of the solid and fluid phases of a porous material and hence make these equations non-homogeneous as well. Subsequently, obtaining an analytical solution of such problems is accompanied with extra difficulties. The effect of constant internal heat generation on the temperature field in the porous region under LTNE was addressed for the first time by Yang and Vafai [17] in the fully developed region of a channel filled with a porous material. They introduced the heat flux bifurcation phenomenon (i.e. different heat flux directions for the two phases at the walls). Their analysis was further extended to the micro-scale by Mahmoudi [7] and Dehghan et al. [18] to find the thermal response and the Nusselt number in the slip-flow regime wherein the temperature jump phenomenon also govern at the walls. Dehghan et al. [18] used the two-equation energy model [21] to define conditions at which the heat flux bifurcation occurs in the slip-flow regime. They also discussed the validity region of the one-equation energy model (i.e. the LTE assumption). Dehghan [8] introduced the heat flux bifurcation (splitting) phenomenon in a channel partially filled with a non-Darcian porous material. He discussed the thermal response of such partially porous-filled channels under the LTNE condition using the two common thermal boundary conditions (models A and B).

The above literature review demonstrates that solving the heat transfer problem in a channel or pipe filled with a porous martial subject to constant wall heat flux boundary condition under LTNE model in the developing region or fully developed region requires the use of thermal boundary condition models (i.e. Model A, B or C (e.g. References [5,15,16])) at the solid wall. To date, it is not clear which two boundary conditions are required at the solid walls since all the existing models are validated against experiment. Hence, the previous numerical (e.g. Reference [6]) and analytical (e.g. Reference [5]) studies provided solutions for the temperature fields in the porous region that are model dependent.

Thus, in the present study, we revisit the problem and examine the thermally developing forced convection heat transfer inside a channel filled with a porous material in the presence of internal heat generations under LTNE condition. It is assumed that the temperature difference between the two phases in the porous region is small such that a linear perturbation analysis can be performed to avoid using models for the constant heat flux thermal boundary conditions at the channel wall (i.e. A, B and C). Therefore, analytic solutions are obtained for the temperature difference between the solid and the fluid phases as well as the Nusselt number in the developing region of the channel. Effects of pertinent parameters including the effective conductivity ratio, $k$, the Biot number, $Bi$, and the internal heat generation parameter, $\omega$, on the Nusselt number, $Nu$, and the thermal entry length, $x_{dev}$, are discussed.

## 2. Mathematical modeling

The problem studied is schematically shown in Fig. 1. A channel with height of 2H and length of $L$ is filled with a porous material. Channel walls are subjected to a constant wall heat flux. Laminar flow with a uniform temperature ($T_i$) enters the channel. Assumptions in the present analysis are as follows:

- 2D channel with impermeable parallel-plates to ensure the two-dimensionality of the problem (e.g. References [5–8]),

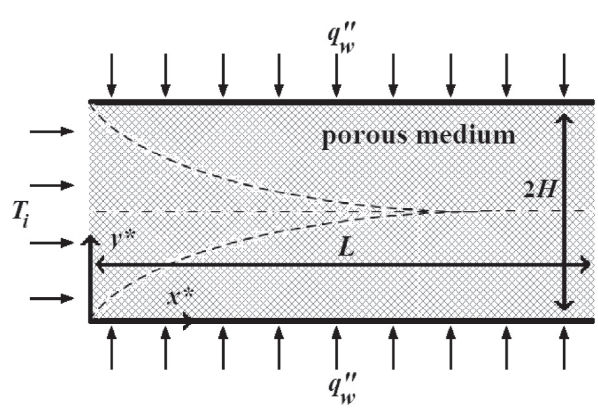


**Fig. 1.** Schematic diagram of the problem.