Electric Power Systems Research 104 (2013) 9–17

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Robust WLS estimator using reweighting techniques for electric energy systems

Eduardo Caro [a,*], Roberto Mínguez [b], Antonio J. Conejo [c]

[a] Universidad Politécnica de Madrid, Madrid, Spain
[b] Universidad de Cantabria, Cantabria, Spain
[c] Universidad de Castilla-La Mancha, Ciudad Real, Spain

ARTICLE INFO

Article history:
Received 31 July 2012
Received in revised form 24 April 2013
Accepted 31 May 2013
Available online 6 July 2013

Keywords:
Weighted least squares
Power system state estimation
Outlier detection
Dependent Gaussian measurements

ABSTRACT

The state estimator is a key tool in the operation of any real-world electric energy system. In this paper, a state estimator based on a weighted least squares model is proposed which is robust against outliers. This algorithm presents two relevant features: robustness that is achieved by readjusting measurement weights, and accuracy that is attained by considering measurement dependencies. The proposed method is tested in the IEEE 57-bus and 118-bus systems and the obtained results are analyzed using Design of Experiments and ANOVA techniques.

1. Introduction

1.1. Motivation

In any real-world electric energy system, the Control Center monitors and controls the functioning of the network in real-time, ensuring operational security. To accomplish this task the Control Center needs to know accurately the actual state of the system (node voltages, power flows, etc.) at any time. These values are estimated by the state estimator (SE).

The state estimator is a mathematical algorithm which computes the most-likely state of the network, given a redundant set of measurements captured from the system. From the statistical point of view, the state estimation algorithm is a nonlinear multiple regression problem, whose parameters to be estimated are those which characterize the network state: node voltage magnitudes and angles.

This estimated state is generally computed using the Maximum Likelihood Estimator, minimizing the weighted sum of the squared residuals (i.e., Weighted Least Squares approach). Once the most-likely state is obtained, the Control Center performs a "bad data detection and identification procedure" to detect and eliminate those measurements whose associated standardized errors are larger than a pre-established tolerance. The statistical tests commonly employed for these tasks are the χ^2-test and the Largest Normalized Residual test, and are well established in the technical literature [1]. Once outliers have been removed, the nonlinear multiple regression problem is solved again, and the final state estimate is obtained.

If outliers are not properly detected or eliminated, the final estimate will be biased, and the Control Center will not have an accurate knowledge of the actual state of the system, leading occasionally to an insecure operation of the network. For this reason, the detection and identification of bad measurements have a notorious relevance in the estimation process. In fact, an adequate and secure control is only achieved in the case that the SE procedure is robust enough to detect and eliminate the presence of corrupt measurements.

Traditionally, the "outlier elimination" problem is solved iteratively by detecting/removing suspected measurements and re-estimating the state disregarding the rejected data. These estimators are based on the weighted least squares, which shows a notable computational efficiency; however the lack of robustness deteriorates significantly their performance in the presence of bad measurements. Specifically, the presence of multiple conforming bad measurements in the measurement set may provoke a "masking effect": good measurements may be rejected whereas corrupted ones may not. This undesirable situation occurs when measurement dependencies are not properly modeled.

1.2. Aim

The aim of this paper is to present a robust state estimator based on a weighted least squares regression, which carries out

* Corresponding author at: C/ José Gutiérrez Abascal, 2,28006 Madrid, Spain. Tel.: +34 913363149.
E-mail address: Eduardo.Caro@upm.es (E. Caro).

0378-7796/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.epsr.2013.05.021

the estimation and the bad data detection/identification processes simultaneously by successively adjusting the weighting matrix and considering the effect of measurement dependencies. The obtained estimate does not require further bad measurement processing algorithms.

1.3. Literature review

The technical literature is rich in references concerning the state estimation problem, for instance, [2] or [1]; and there is a significant number of references on outlier detection: [3–11]. The previous works are focused mainly on the area of least squares linear regression. Other statistical models and estimation methods, such as reweighed techniques [12–15], non-linear methods [16], variance-varying models [17], or some robust estimators [18–20] have received comparatively less attention. Nevertheless, [21] report successful results from the application of the reweighted least deviation method developed by [14], to detect data related to hurricanes and typhoon on wave hindcast databases.

However, no so many works address the power system WLS estimator using adjusted measurement weights. The pioneering work reported in [22] proposes a method for readjusting the measurement variances based on the residuals of previous estimations. Ref. [23] develops this approach, improving the computational efficiency and ensuring mathematical convergence. Work [24] propose an iterative reweighted least-squares estimator that is based on Givens Rotations and improves the robustness against outliers.

In [25], the weights of the WLS estimator are artificially manipulated, leading to a more robust estimator with the properties of the weighted least absolute value approach. Recently, in [26], the WLS regression is addressed using estimated weights based on the measurement variances.

All the aforementioned works consider that the measurement covariance matrix is diagonal. However, recent works [27,28] show that this matrix is generally non-diagonal. Thus, the reweighting techniques previously proposed in the technical literature can be improved, since such techniques cannot deal with measurement dependencies.

1.4. Contribution

The contribution of this paper is threefold:

- First, it provides a mathematic procedure that allows applying a reweighting estimation technique (originally designed for diagonal covariance matrices) to a non-diagonal estimation problem.
- Second, an iterative state estimator is proposed, showing both robustness against outliers and computational efficiency. Specifically, it requires significantly less time that similar methods proposed in the technical literature [29].
- Finally, Design of Experiments and ANOVA procedures are used to compare the performance of the proposed method with statistical rigor.

1.5. Paper organization

The rest of this paper is organized as follows. Section 2 develops and formulates the Reweighted Least Squares Estimator considering measurement dependencies. Section 3 applies the Design of Experiments and ANOVA procedures to the considered estimation problem. Section 4 provides and analyzes results from four realistic case studies. Finally, Section 5 provides some relevant conclusions.

2. Dependent state estimation model

Any state estimator can be formulated as a nonlinear multiple regression problem, where the unknown parameters are the node voltage magnitude and angle of every node, represented by V_i and θ_i, respectively. These two sets of variables form the state vector $\boldsymbol{x} = [\boldsymbol{V}^T \quad \boldsymbol{\theta}^T]^T$. There are n state variables. The unknown true state is represented by $\boldsymbol{x}^{\text{true}}$.

The unknown parameters are estimated using the information provided by observations $\{z_1, \ldots, z_m\}$. These observations are captured from the system using measuring devices, and are related with $\boldsymbol{x}$ by means of a multifunctional vector $\boldsymbol{h}(\boldsymbol{x})$. Depending on the measurement type, the functions $h_i(\boldsymbol{x})$ differ. Expressions of functions $h_i(\boldsymbol{x})$ are well-established in the technical literature [1].

The error terms used throughout this paper are defined below:

- *Measurement residual*: difference between the measurement z_i and the function $h_i(\boldsymbol{x})$ evaluated at the optimal state $\hat{\boldsymbol{x}}$,

$$\text{Residual}_i = z_i - h_i(\boldsymbol{x})|_{\boldsymbol{x}=\hat{\boldsymbol{x}}} = z_i - h_i(\hat{\boldsymbol{x}}) = r_i. \quad (1)$$

- *Measurement error*: difference between the measurement z_i and the function $h_i(\boldsymbol{x})$ evaluated at any state $\boldsymbol{x}$,

$$\text{Error}_i = z_i - h_i(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}} = z_i - h_i(\boldsymbol{x}) = e_i. \quad (2)$$

- *Metering error*: difference between the measured value and the unknown "true" value,

$$\text{Metering}\,error_i = z_i - h_i(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}^{\text{true}}} = z_i - h_i(\boldsymbol{x}^{\text{true}}) = z_i - z_i^{\text{true}}. \quad (3)$$

Note that the term "measurement residual" is solely used in the case of comparing measurement value z_i with the function $h_i(\cdot)$ evaluated at the optimal state $\hat{\boldsymbol{x}}$. Similarly, the term "measurement error" is solely employed for comparing the measured value z_i with the function $h_i(\cdot)$, evaluated at any state $\boldsymbol{x}$.

Measurement errors have been traditionally modeled as an independent unbiased Gaussian-distributed random variable. The factual metering infrastructure within substations results in significant statistical correlations between measurement errors. Works [27] and [28] numerically show that these correlations are significant, and its consideration may improve the quality of the final estimate. Therefore, hereafter measurement errors are assumed to be dependent Gaussian-distributed unbiased random variables. The dependence structure is modeled by means of positive-definite non-diagonal variance-covariance matrix $\boldsymbol{C}_z$, which can be easily computed using the Point Estimate method [28].

As it is customary in the technical literature, all measurements are considered synchronous. This is reasonable since in steady state power system magnitudes change very slowly with respect to the time needed to transfer measurements to the EMS from Remote Terminal Units (RTUs) or Phasor Measurement Units (PMUs).

2.1. State estimation

Given the previous assumptions, the estimation of the state variables are obtained by minimizing the weighted sum of squared measurement errors of the multiple nonlinear regression model, leading to a nonlinear optimization problem:

$$\underset{\boldsymbol{x}}{minimize}\, J = [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^T \boldsymbol{C}_z^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \quad (4a)$$

subject to

$$\boldsymbol{c}(\boldsymbol{x}) = \boldsymbol{0} \qquad \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0} \quad (4b)$$

where the scalar J is the objective function and $\boldsymbol{c}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are the equality and inequality constraints modeling zero-injections nodes and physical operating limits, respectively. Note that matrix $\boldsymbol{C}_z$ is