Electric Power Systems Research 89 (2012) 1–10

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies

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ARTICLE INFO

Article history:
Received 4 July 2011
Received in revised form 29 January 2012
Accepted 30 January 2012
Available online 6 March 2012

Keywords:
Phasor measurement unit (PMU)
Optimal placement
Power system observability
Measurement security
Integer programming

ABSTRACT

The application of phasor measurement units (PMUs) in power systems is increasing because of their advantages such as the capability for online state estimation and improvements in the speed of control, and protection systems. In this paper, we propose a new method using binary integer linear programming for the optimal placement of PMUs to guarantee full observability of a power system as well as maximizing the measurement redundancy. Moreover, the problem of the optimal placement of these units in the case of a single PMU loss or single line outage is investigated. A practical limitation is also considered on the maximum number of PMU channels, in the proposed formulation. In all of the investigations, the effect of zero-injection buses in the power system was considered. The efficiency of the proposed method was demonstrated in different conditions. The method was applied to several IEEE standard test systems, i.e., the 14-, 30-, 39-, 57-, and 118-bus systems, and in two very large-scale systems, i.e., 2383- and 2746-bus systems. The simulation results verified the acceptable performance of the proposed method.

1. Introduction

Currently, rapidly increasing power demands, disproportionate growth of power generation and transmission systems, power system restructuring, and other factors have overloaded the existing electrical networks and subsequently decreased the stability margin of these networks [1,2]. In such circumstances, to ensure the stable and proper operation of the system, a precise measurement and monitoring of the system states are required. This monitoring was conventionally performed by utilizing the supervisory control and data acquisition (SCADA) system, in which state estimation is derived based on measurements that are not usually synchronized. Therefore, a steady state or, in the most optimistic situation, a quasi-steady state will be obtained for the power system. Hence, the system operator has no access to the dynamic state of the system, which is required to maintain the system in the normal condition. To overcome this limitation in the SCADA, the wide-area monitoring, protection and control (WAMPAC) system has been employed, in which phasor measurement units (PMUs) are considered the basic components [2]. These units, which are synchronized with clock signals from global positioning system (GPS) satellites, are able to provide synchronized measurements [1,2]. When these units are installed on a system bus, the phasor of the bus voltage can be measured as well as the phasor of the line currents emanating from that bus. Hence, the voltage phasor of adjacent buses can be calculated using Kirchhoff's laws in the steady-state condition. Therefore, it is not necessary to install these units on all of the system buses to control or estimate the system states [2,3]. Furthermore, installation of the PMUs on all of the system buses is impossible because of their high cost and the lack of communication facilities [3]. Thus, one of the important issues is to find the optimal number and location of PMUs according to the desired objectives.

The main goal is therefore to minimize the number of required PMUs to be installed in the power system while maintaining full observability of the system. To date, several methods have been employed to solve this optimization problem. These methods can generally be divided into conventional methods and heuristic optimization algorithms.

In conventional methods, the optimal placement of PMUs is expressed as an integer linear programming problem [4–10]. Hence, the proper definition of constraints that guarantee the satisfaction of the desired objective plays a key role in achieving the optimal solution. Therefore, the differences between these methods lie in the formulation of the required constraints. In [4], with the aim of full network observability, the constraints of the optimal placement problem are formulated as a set of nonlinear inequalities. In [5], after changing the system topology, the constraints are derived as a set of linear inequalities for the reconfigured system. In the formulations presented in [6–10], the constraints are introduced as a set of linear inequalities, without any reconfiguration in the system topology. Despite all of these differences,

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doi:10.1016/j.epsr.2012.01.020

the common feature of these methods is the small solution time for large-scale systems. We note that each of these methods can achieve an optimal solution for the PMU placement problem to serve a specific purpose. For example, among the prior studies, the minimum number of PMUs needed to ensure that the power system is fully observable was obtained in [9,10]. However, in [6], a smaller number of PMUs are presented to guarantee the full observability of the power system in the case of a single PMU loss. Therefore, there remains the need for a generalized method for determining the optimal number and locations of PMUs to satisfy these goals in various cases.

Among the heuristic optimization algorithms, simulated annealing [3], binary search [11], Tabu search [12], genetic algorithm [13], particle swarm optimization [14–16], ant colony optimization [17], immunity genetic algorithm [18] and immunity particle swarm optimization [19] have been developed. Unlike conventional methods, in these methods, there is no need to define a set of constraints to achieve the desired goal. A special property of these methods is that the algorithm parameters can be tuned flexibly so as to obtain the best solution. Using these methods for the PMU placement problem, the number of unobservable buses is considered a component of the objective function. Hence, by minimizing this term, full observability of the power system is guaranteed [3,13–15,17–19]. Among these previous works, the minimum number of required PMUs to achieve the full observability of the power system is presented in [14,15,18], and this number is equal to that obtained in [9,10] using the conventional methods. In comparison to conventional methods, the utilization of heuristic methods causes increase in solution time for very large-scale power systems [18].

In this paper, a new method is presented for the optimal placement of PMUs utilizing an integer programming technique. This method is able to determine the minimal number and optimal locations of PMUs in order to provide the full network observability as well as maximizing the measurement redundancy in normal operation and also in the case of a single PMU loss or a single line outage. The proper objective function and the required constraints to ensure the achievement of these objectives are described without changing the system topology. In addition, the proposed formulation is extended to consider a practical limitation on the maximum number of PMU channels.

In the next section, the formulation of the optimal placement of PMUs with the aim of full network observability is presented. In Section 3, the necessary constraints to maintain full network observability given the occurrence of a single PMU or line outage are described. Finally, the performance of the presented formulation is assessed using several IEEE standard systems. The simulation results show the ability of the proposed method to yield the optimal placement of these units under different conditions.

2. Problem formulation in normal operation of the power system

To find the optimal locations of PMUs, it is sufficient to know the system topology and the type of system buses. The system bus connections are displayed using a system connectivity matrix. This matrix shows the interconnection of buses by transmission lines. Here, denoting A as the connectivity matrix and N as the number of system buses, A forms an $N \times N$ matrix with entries defined as follows [4–10,16]:

$$[A]_{ij} = a_{ij} = \begin{cases} 1, & \text{if } \; i = j \\ 1, & \text{if buses } \; i \; \text{ and } \; j \; \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The discrete nature of the optimal PMU placement problem requires the vector X to be defined as follows [4–10,16]:

$$[X]_i = x_i = \begin{cases} 1, & \text{if a PMU is installed at bus } \; i \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where each entry of this vector shows the status of the installation of a PMU on each bus. Because the aim of solving the optimal placement of PMUs is to find the minimum number of buses on which these units must be installed, the objective function of the problem can be written as follows [7–9]:

$$\min \sum_{i=1}^{N} x_i \tag{3}$$

In the above objective function, the cost of all PMUs is assumed to be equal. The cost of a PMU depends on several factors; such as the number of measuring channels, CT and PT connections, power connection, ground connection, and the GPS receiver. However, what really distinguishes between different PMU costs is the number of channels, because the remainder items are the same for all PMUs [6]. It is obvious that a PMU with more channels is costlier than that of with less number of channels. To consider unequal cost for PMUs, x_i in (3) should be replaced by $c_i x_i$, where c_i is the cost of installed PMU at the ith system bus [4–6,9,10]. The cost of a PMU which is installed on a bus with only one incident line can be set to 1 per-unit. For each additional incident line, the cost will be increased with an incremental factor α. A reasonable selection for α is 0.1 [6].

Minimization of the number of used PMUs by the objective function (3) may leads to various arrangements of PMUs with the same numbers. The question is this: "which arrangement is the best one to be installed?"

In this paper, maximizing the measurement redundancy throughout the power system is considered as the objective in the problem of optimal measurement placement. In this way, the following objective function is selected:

$$\min \left(\sum_{i=1}^{N} c_i x_i \right) + w \left(\sum_{i=1}^{N} m_i - \sum_{i=1}^{N} f_i \right) \tag{4}$$

where $w \in \Re$ is a weighting factor which is selected such that the two components of the objective function could be comparable in terms of magnitude, m_i represents the maximum number of times that the ith bus can be observed ideally (i.e., the number of its incident lines plus one), and f_i represents the number of times that the ith bus is observed by the set of installed PMUs obtained in X.

The second term in (4) indicates the difference between the sum of ideal and actual number of times that each bus of the system is observed. The maximum redundancy will be gained by minimizing this difference, subsequently.

Proper definition of the constraints that ensure the full observability of the system is the key to the solution of the optimal PMU placement problem. This definition changed according to the existing conditions of the system such as the inclusion or exclusion of zero-injection buses. Hence, in the following, we develop a method for determining the constraints of the optimization problem for each case. To facilitate this discussion, a 7-bus system is used as an example, as illustrated in Fig. 1. Here, the solid circles are system buses and zero-injection buses are distinguished from other buses by dots beside them (here, buses 3 and 5).

2.1. Problem constraints without considering zero-injection buses

In the proposed method, we assume that a PMU installed on a bus has a sufficient number of input channels to measure both the voltage phasor of the bus and the current phasors of all lines