Electric Power Systems Research 116 (2014) 136–146

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Unified boundary and probabilistic power flow

A. Mohapatra*, P.R. Bijwe, B.K. Panigrahi

Department of Electrical Engineering, Indian Institute of Technology Delhi, New Delhi 110016, India

ARTICLE INFO

Article history:
Received 23 January 2014
Received in revised form 22 May 2014
Accepted 24 May 2014
Available online 19 June 2014

Keywords:
Uncertainty
Power flow
Boundary
Probability
Optimal multiplier
Line outage simulation

ABSTRACT

Different types of uncertainties exist in system data. Outages, errors in load forecasts and renewable generations are generally represented as probabilistic uncertainties. Load model coefficients and network parameters, on the other hand, are best represented as interval uncertainties. Irrespective of the nature of these uncertainties, all of them need to be considered in an integrated manner for proper system analysis. This paper tries to fulfill this precise need. By utilizing the synergy of boundary and probabilistic power flow algorithms, development of efficient line outage simulation and use of constant Jacobian approach, the computational burden has been kept to a manageable level. The proposed approach can be used for both transmission and distribution systems. Results for two transmission and one distribution systems have been obtained with various types of uncertainties. Validation of results has been done through the Monte Carlo Simulations (MCS).

1. Introduction

Power flow is an important tool in day-to-day system operations and planning. In practice, data is never available with complete certainty. Power flow is also no exception. Hence, uncertainty plays an important role in this analysis. The type of model for handling uncertainties depends on the value of information available to represent a specific data.

Historical records indicate that every contingency or outage has a probability of occurrence. Security enhancement is done using optimal use of preventive, corrective and emergency actions which may require changing the base case operation. Sometimes, costly but very useful, load side reserves are used. All these involve substantial costs. Hence, not just the severity of the event, but the probability of the same is always useful in deciding the action plan. It is, thus, best represented in the form of a probability distribution [1]. Typically, binomial distribution has been used to represent the outages. Such representation has been useful for reliability assessment, security analysis and available transmission capacity determination [2]. Errors in load forecasts and renewable generations have also been represented as probabilistic uncertainties. Normal distribution has been used for representing load forecast errors while Weibull distribution has been used for representing uncertainty in wind power generation.

Probabilistic representation is possible only when the uncertain data is random and repetitive in nature. Hence, uncertainties in data such as load model coefficients and network parameters, cannot be represented in probabilistic sense. Also, if only ranges of output variables are of interest, these can be easily represented by use of fuzzy membership functions, of which interval representation is a typical type. Thus, load model coefficients and line parameters can be easily represented in terms of intervals or boundary values. Sometimes, the same may also be true even for load forecast errors [3] when sufficient statistical data is unavailable. Hence, representation of each type of data in its most suitable way is necessary depending on the availability of associated statistical variations. Approximations made in representing all data either in interval or probability sense may not give the desired solutions.

Literature includes myriad of papers on this uncertainty analysis. This can be broadly classified into: first, stochastic power flow of which probabilistic power flow (PPF) [4–12] is most common and second, fuzzy power flow (FPF) [3,13–18]. Two notable methods of PPF are point estimate method (PEM) [6–8] and cumulant method (CM) [11,12]. PEM has variants of which 2PEM (two point estimate) is the simplest. PEM PPF allows for nonlinear analysis while CM could only be used for linear analysis. Boundary power flow (BPF) [3,14,18] is a specific sensitivity based variant of FPF. Both, PEM PPF and BPF are computationally intensive and thus the computational burden of a unified boundary and PPF could be immensely huge.

Approaches for handling outage type uncertainties in power flow have also been proposed [5,19–22]. However, most of them have been on the use of linear power flow equations. Also, in practical systems, effects of loss of a line or a unit must be economically

---

[*] Corresponding author. Tel.: +91 11 26591046.
E-mail addresses: abheejeet_ee@student.iitd.ac.in, abheejeet911@gmail.com (A. Mohapatra).

http://dx.doi.org/10.1016/j.epsr.2014.05.015
0378-7796/

shared by other generating units by means of distributed slack power flow, or also known as automatic generation control power flow (AGCPF) [23].

Combined probabilistic and fuzzy approaches for power system reliability and risk assessment exist in the literature [1,24–29]. However in most of these, use of probabilistic or fuzzy modeling is mostly for constraint satisfaction or for minimization of some objective, and not for the representation of the uncertain system data itself. Li et al. [27] present a method for studying the combined effect of fuzzy model for peak load with probabilistic distribution for load curve on power system reliability. Outages are modeled using MCS. Pourahmadi-Nakhli et al. [30] propose a nonlinear combined power flow (MCS + FPF [18]) for distribution systems with hybrid fuzzy/probabilistic models for system loads and renewable generations. MCS is used for dealing with probabilistic uncertainties. However, use of MCS or enumeration techniques leads to higher computation time in terms of repeated power flow executions [19]. It is thus expected that the approaches in [27,30] may be computationally inefficient. Also, these approaches have not considered the uncertainties in load model coefficients and network parameters.

Hence, power flow solution must be obtained with all uncertainties considered together. The first motivation here is to fulfill this need. Although, literature shows that hybrid approaches exist, however, to the best of authors' knowledge, no efficient attempt has been made to solve a unified boundary and PPF. Also, PPF and BPF techniques have been separately developed. Bringing them together has the unwelcome prospect of making this exercise computationally unviable. Hence, the second motivation is to develop an efficient strategy which utilizes the complimentary synergies of both. In addition, a new accurate and efficient nonlinear AGCPF based line outage simulation methodology has been developed. The entire methodology uses a constant Jacobian to further improve the computational efficiency. Such a unified approach can be used to assess adequacy indices based on the worst possible value of a desired output along with an associated change in probability of violating its operational limit. These indices play an important role in system operation and planning [1]. The proposed unified approach can be used in both long-range resource planning and short-range operations. The extent of uncertainty in a given time frame and the computational burden admissible for the analysis of the same will decide whether it can be used in real time. It can definitely be used for day ahead scheduling and may be an hour or few minutes prior to real time operations.

The rest of the paper is thus as follows: AGCPF, BPF and PEM PPF are briefly discussed in Section 2. Section 3 presents the constant Jacobian based AGCPF for line outage simulation. The approach for unified power flow is discussed in Section 4. Results for the IEEE 30 and IEEE 118 bus systems and a 69 node distribution system are given next, which prove the efficacy of the proposed approach. Relevant conclusions are summarized in Section 6.

2. Preliminaries

2.1. Conventional AGCPF [23]

Whenever load/generation changes occur or in the event of occurrence of an outage, conventional power flow dumps the mismatch on the slack bus which is impractical. AGCPF allows practical sharing of the mismatch on the participating generators, as per the given participation factors [23]. This is best suited for the purpose of unified power flow. The deterministic AGCPF equations can, thus be stated as:

$$\begin{aligned} &\mathbf{P}^{sp} + \boldsymbol{\beta} P_{mis} - \mathbf{P} = 0 \\ &\mathbf{Q}^{sp} - \mathbf{Q} = 0 \end{aligned} \tag{1}$$

where $\mathbf{P}^{sp}$ and $\mathbf{Q}^{sp}$ are specified real and reactive powers, respectively. $\mathbf{P}$ and $\mathbf{Q}$ are respective calculated injections. $\boldsymbol{\beta}$ is vector of participation factors such that $\sum_j \boldsymbol{\beta}_j = 1$ and P_{mis} is the real power mismatch. In general, real power equations are solved for all buses while reactive power equations are solved only for load buses. These can be solved by the fast decoupled method [31] or the conventional Newton's method. Newton's method has been used here for solving AGCLF (1) and thus the coupled kth iteration update equation is as follows:

$$\begin{bmatrix} \Delta\delta^k \\ \Delta\mathbf{V}^k \\ \Delta P_{mis}^k \end{bmatrix} = \mathbf{J}_{pk}^{-1} \begin{bmatrix} \mathbf{P}^{sp} + \boldsymbol{\beta} P_{mis} - \mathbf{P}^k \\ \mathbf{Q}^{sp} - \mathbf{Q}^k \end{bmatrix} \tag{2}$$

where $\mathbf{V}$ and δ are voltage magnitude and angle vectors and $\mathbf{J}_{pk}$ is the kth iteration Jacobian in polar co-ordinates.

Handling bus switchings due to reactive power violations at generator buses is done through the use of compensation technique [32]. Thus in (1), reactive power equations for generator buses are also present with masking shunts as in [32]. Hence, the number of equations in (1) are $2N$ for a N bus system. It is to be noted that in the discussions that follow, AGCPF shall be referred as the crisp power flow. BPF using this crisp power flow is briefly discussed next.

2.2. Boundary power flow [3,14]

BPF is a special case of FPF, in which uncertainties are assumed to be in an interval or boundary with unity membership. Typical uncertainties of this type could be in network parameters, load model coefficients and loads and generations if sufficient statistical data is unavailable.

BPF begins with a crisp power flow solution with inputs $\mathbf{U}$ at their central values. Then, for a given range of $\mathbf{U}$, boundary values of output variables $\mathbf{Z}$ are obtained as [3,14]:

$$\mathbf{Z} = \mathbf{Z}_e + \mathbf{L}[\mathrm{U}_{sp} - \mathbf{U}_e] \tag{3}$$

where U_{sp} is vector of interval specifications and $\mathbf{U}_e$ is the current estimate of $\mathbf{U}$. $\mathbf{L}$ is the sensitivity of $\mathbf{Z}$ with respect to $\mathbf{U}$. $\mathbf{Z}_e$ is current estimate of $\mathbf{Z}$ evaluated at $\mathbf{U} = \mathbf{U}_e$. Boundary values of ith output variable Z_i, are obtained as:

$$Z_i = Z_{ei} + \sum_{j=1}^{n} L_{ij}(\tilde{U}_{spj} - U_{ej}) \tag{4}$$

where n is cardinality of $\mathbf{U}$. In BPF, an appropriate extreme value of $\tilde{U}_{spj}$ is selected, depending on whether a maxima or minima of Z_i is desired. If $Z_i^{\min}$ is of interest, then $\tilde{U}_{spj} = \tilde{U}_{spj}^{\min}$ if L_{ij} is positive and $\tilde{U}_{spj} = \tilde{U}_{spj}^{\max}$ if L_{ij} is negative. Similar logic holds true if $Z_i^{\max}$ is of interest. It is important to note that role of (4) is only for input selection and not for calculation of Z_i. With the selected input, actual Z_i is evaluated from the usual crisp power flow. This completes one iteration of BPF. The entire process must be repeated for all boundary values of all variables of concern.

If U_j is a load or a renewable generation, then L_{ij} is (i, j)th element of $\mathbf{S}\mathbf{J}_{pe}^{-1}$ where $\mathbf{S}$ is sensitivity of Z_i with respect to states and $\mathbf{J}_{pe}$ is current estimate of Jacobian. The BPF remains same as discussed