Applied Thermal Engineering 60 (2013) 325–334

Contents lists available at SciVerse ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

The critical and crossover radii on transient heating

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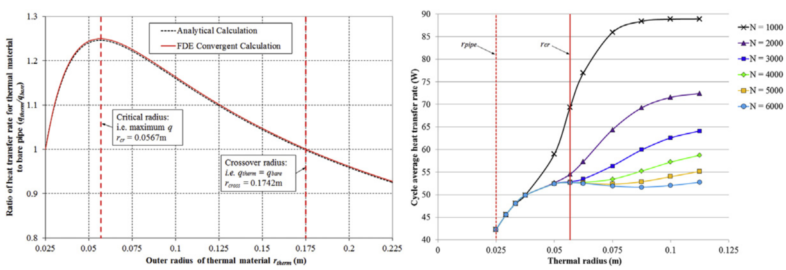
HIGHLIGHTS

- Steady-state critical and crossover radii may be applicable to transient systems.
- This requires that the heating-cooling time be sufficiently high.
- Stabilisation time is constant for a given thermal material and geometry.
- The critical radius stabilises more quickly than the crossover radius.
- Prior to stabilisation the critical and crossover radii are ill-defined.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 16 January 2013
Accepted 27 June 2013
Available online 11 July 2013

Keywords:
Critical radius
Crossover radius
Heat transfer
Transient heat transfer
Optimization
FDE

ABSTRACT

For cylindrical and spherical heat transfer systems, it is imperative that the radius of thermal material is matched to the associated heat transfer objective. For systems that intend to *maximize* heat transfer, the critical radius defines the optimal radius for a specific scenario. For systems that intend to *minimize* heat transfer, the crossover radius defines the radius required to achieve an equal heat transfer rate to the un-insulated scenario; with any further increase in radii resulting in a monotonic reduction in the associated heat transfer rate. The critical radius is well defined for the steady-state scenario. In recent literature, the steady-state crossover radius has also received attention; however, the literature does not provide clarity for the transient scenario. This work overcomes this identified limitation by quantifying the crossover and critical radii of a transient cylindrical system, allowing novel conclusions to be drawn between the steady-state and transient scenarios. In particular, this work identifies that the cycle-average heat transfer rate can stabilize to a quasi-static value in response to transient heating.

1. Introduction

The heat transfer rate for planar, cylindrical and spherical systems is well understood for steady-state thermal loading. For planar systems, the resistance to heat transfer increases monotonically with increasing insulation thickness. For cylindrical and spherical systems, the resistance to heat transfer varies non-linearly, and it is therefore of significant importance that the radius of thermal material be matched to the associated heat transfer objective.

Transient thermal loading of cylindrical or spherical systems is an important design scenario for applied thermal engineering. For example, all thermal systems are in a transient state from when a heat load is initially applied to the time of temperature stabilisation; and, thermal systems are also often transient during normal operation. For example, heating or cooling by cylindrical piping is often either: applied in a transient manner, or is applied at near constant temperature to a system with a transient heat load. A practical example of the former case is domestic hot water that is intermittently delivered by piping, resulting in transient heat transfer. An example of the latter case is cooling water that is delivered to a moulding die at a constant inlet temperature; however, fluctuation of the die temperature through the manufacturing process results in transient heat transfer.

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http://dx.doi.org/10.1016/j.applthermaleng.2013.06.052

Nomenclature

α	thermal diffusivity [m^2/s]
A	area [m^2]
Bi	Biot number
D	duty cycle [s/s]
d	diameter [m]
h	convective heat transfer coefficient [W/m^2 K]
i	space-domain subscript $\in [0,n]$
k	thermal conductivity [W/m K]
L	length [m]
N	number of cycles
P	cycle period [s]
R	thermal resistance [K/W]
r	radius [m]
T	temperature [K, °C]
T_∞	ambient temperature [K, °C]
∇T	temperature gradient [K/m, °C/m]
t	time [s]
q	heat transfer rate [W]
L_C	characteristic length [m]
l	time-domain subscript $\in [0,\infty]$
q'	heat flux [W/m^2]
$\hat{q}$	cycle-average heat transfer rate [W]

For steady-state systems that intend to maximize heat transfer, the critical radius defines the optimal radius for a specific thermal material. However, for systems that intend to minimize heat transfer, the critical radius does not provide robust design guidance. For systems that intend to minimize heat transfer, the crossover radius defines the minimum radius required to achieve equal heat transfer to the un-insulated case; where any further increase in radius results in a monotonic reduction in the associated heat transfer rate.

The crossover radius has received attention in recent literature [1,2], due to its importance in design optimisation for systems that intend to minimize heat transfer; however, the results are only developed for the steady-state scenario. This study investigates the crossover and critical radii of a transient cylindrical system. In particular, a method for evaluating the transient heat transfer associated with cylindrical systems is developed, and used to draw novel conclusions between the steady-state and transient heat transfer scenarios. These conclusions provide previously unavailable design guidance for the optimisation of cylindrical systems subject to transient thermal loading.

In addition to providing direct design guidance for scenarios that are known to be transient, the contributions of this work allow reverse engineering of a given cylindrical heat transfer scenario, to identify the range of design scenarios for which the system response will be either transient, or quasi-static. Furthermore, the computational cost of the proposed method may be relatively high; this work identifies opportunities to reduce the associated computational cost.

1.1. Heat transfer modes

A temperature gradient within a media results in heat transfer by the mechanisms of conduction, convection and radiation [3]. In this work, the driving temperature difference will be sufficiently low that the effect of radiation will be ignored and heat transfer will be assumed to occur by the modes of convection and conduction.

Conduction occurs by molecular vibration from more energetic particles to less energetic particles in the absence of bulk matter transfer [4]. Fourier's law defines the rate of heat conduction, q_{cond}, as a function of the associated area, A, thermal conductivity, k, and temperature gradient, ∇T (Eq. (1)).

$$q_{\mathrm{cond}} = -kA\nabla T \tag{1}$$

The heat flux is defined as the heat transfer rate per unit area:

$$q' = q/A \tag{2}$$

Convection occurs due to the combined effects of advection and diffusion due to fluid motion as expressed by Newton's law of cooling (Eq. (3)); where the associated heat transfer rate q_{conv} is a function of the convective heat transfer coefficient, h, associated area, A, and the temperature difference between the surface temperature, T_s, and ambient temperature, T_∞ [4].

$$q_{\mathrm{conv}} = hA(T_s - T_\infty) \tag{3}$$

The resistance to heat transfer is defined as the ratio of the driving potential to the resulting transfer rate (Eq. (4)).

$$R = \Delta T/q \tag{4}$$

1.2. Steady-state heat transfer of cylindrical systems

In the presence of constant boundary conditions, and in the absence of internal heat generation, a temperature field will stabilize to a steady-state condition. The thermal resistance to heat transfer for one-dimensional conduction and convection across a cylinder shrouded by thermal material is given by Equations (5) and (6), respectively [3]. Where r_{cyl} is the cylinder radius, r_{therm} is the outer radius of the thermal material, L is the cylinder length, and k_{therm} is the thermal conductivity of the thermal material (Fig. 1).

$$R_{\mathrm{cond}} = \frac{\ln(r_{\mathrm{therm}}/r_{\mathrm{cyl}})}{2\pi L k_{\mathrm{therm}}} \tag{5}$$

$$R_{\mathrm{conv}} = \frac{1}{2\pi L h r_{\mathrm{therm}}} \tag{6}$$

According to Equations (5) and (6), the thermal resistance between a cylinder and the ambient environment is:

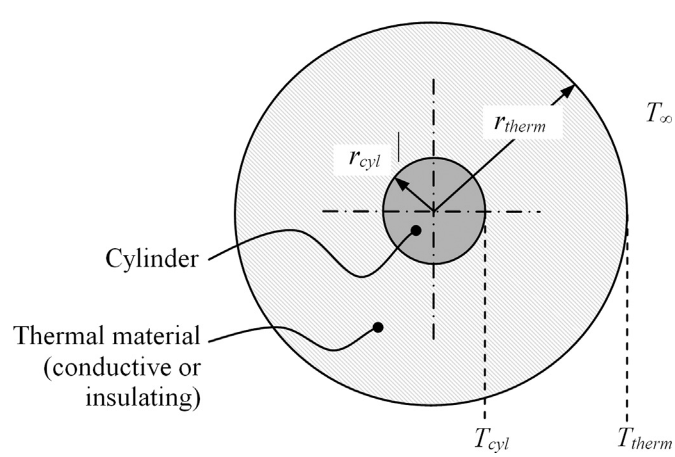

Fig. 1. Cylindrical heat transfer system.