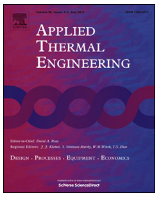
Applied Thermal Engineering 60 (2013) 387–396

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng



Lumped parameter model for one-dimensional melting in a slab with volumetric heat generation

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HIGHLIGHTS

- Lumped model for one-dimensional melting with volumetric heat generation.
- Two-point Hermite approximations for integrals were employed.
- The model can be applied to high Biot number for a wide range of heat generation rates and Stefan number.
- Excellent agreement with available analytical or numerical solutions was achieved.
- The model can be applied for analysis of thermal energy storage system with phase change material.

ARTICLE INFO

Article history:
Received 13 January 2013
Accepted 15 July 2013
Available online 24 July 2013

Keywords:
Melting
PCM
Lumped model
Heat generation
Stefan problem
Biot number

ABSTRACT

In this study, a lumped parameter model was developed for one-dimensional heat conduction with melting of a phase change material (PCM) slab with volumetric heat generation. Two types of boundary conditions were considered: (a) adiabatic condition at the left side and isothermal condition at the right side and (b) isothermal condition at the left side and convective condition at the right side. The lumped model was obtained through two-point Hermite approximations for integrals. The two-side corrected trapezoidal rule ($H_{1,1}$ approximation) was employed in the averaged temperature integrals for both the liquid and solid phases during melting process of the slab, and the plain trapezoidal rule ($H_{0,0}$ approximation) was used to estimate the heat fluxes. For the melting problem with the boundary conditions (a), the lumped model results were verified by the analytical solution of melting in half-plane without internal heat source. Case study was performed to investigate the effect of the volumetric energy generation and the Stefan number on the instantaneous interface position. For the melting problem with the boundary conditions (b), the lumped model results were verified by the enthalpy method solution. Transient heat condition of the PCM slab was analyzed to illustrate the applicability of the proposed lumped model, with respect to different values of the volumetric energy generation, the Stefan number, the Biot number, the liquid-to-solid thermal conductivity ratio and the boundary temperature at the left side. Excellent agreement with available analytical or numerical solutions was achieved.

1. Introduction

In recent years, the application of phase change materials (PCMs) for thermal energy storage has attracted increasing interests due to the considerable latent heat during the phase change process. For the accurate and effective estimation of the PCM's heat storage capability, it is crucial to develop suitable approaches to solve a moving boundary problem (the so-called Stefan problem). In nuclear engineering applications, there are special interests in one-dimensional melting problems which are important in understanding and predicting fuel element behavior under severe accident conditions, such as Three Mile Island (TMI) and Fukushima.

One-dimensional phase change heat conduction problems have been extensively investigated by different analytical, numerical and experimental methods, which can be classified into three major groups based on the geometries of the body, viz., (i) those described by Cartesian coordinates, (ii) those described by cylindrical coordinates and (iii) those described by spherical coordinates. Voller

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http://dx.doi.org/10.1016/j.applthermaleng.2013.07.018

Nomenclature

Bi	Biot numbers
c_p	specific heat, J/kg K
h	convective heat transfer coefficient, W/m^2 K
k	thermal conductivity, W/m K
L	latent heat of melting, J/kg
$\dot{q}$	volumetric heat generation, W/m^3
Q	dimensionless volumetric heat generation
s	solid–liquid interface location
Ste	Stefan number
t	time, s
T	temperature, K
x	space coordinate, m

Greek letters

α	thermal diffusivity, m^2/s
η	dimensionless solid–liquid interface location
θ	dimensionless temperature
κ	liquid-to-solid thermal diffusivity ratio
λ	liquid-to-solid thermal conductivity ratio
ξ	dimensionless space coordinate
ρ	density, kg/m^3
τ	dimensionless time

Subscripts

a	boundary conditions (a)
av	average
b	boundary conditions (b)
e	environment
i	initial
l	liquid
ref	reference
s	solid

and Cross [1] presented an explicit finite difference (FD) scheme for the enthalpy formulation of one-dimensional freezing problems, and also developed an implicit FD algorithm of the Stefan problems with internal heat generation and melting temperature defined by a mushy region, simultaneously. Cheung et al. [2] studied numerically the process of freezing and melting occurring in a heat-generating slab bounded by two semi-infinite cold walls, and solved the dimensionless governing equations using the method of collocation with Hermite splines as approximating functions and Gaussian quadrature points as the collocation points. Wang and Ma [3] performed a one-dimensional analysis, by heat balance integral method, of freezing in a slab with convective cooling boundary condition, and carried out an experimental modeling of freezing heat transfer with convective boundary condition using Peltier devices. Sahin and Dincer [4] presented an approximate analytical solution for the temporal location of moving solid–liquid interface of a phase-change process taking place in parallel plate channels, which were subjected to convective boundary condition at one side and constant heat supply at the other side. Using a quasi-steady approximation, Jiji and Gaye [5] examined analytically one-dimensional solidification and melting of a slab with uniform volumetric energy generation, and applied the results to two examples: solidification of a nuclear material and melting of ice. Bai et al. [6] investigated the cooling process of crystalline plastics in a slab injection mold, and gave an exact solution for the temperature distribution and the rate of phase change, which was verified by the measurements of the temperature profiles of the polymer slab mold instrumented with thermocouples. With the perturbation technique, Yu et al. [7] studied the planar solidification with time-dependent heat generation in a semi-infinite plane, where the results of the proposed method were validated by the exact solution of the classical Stefan problem without heat generation. Jin and Zhang [8] analyzed the thermal performances of the two-layer PCM floor with different melting temperatures for each layer, which were used to store heat or cold energy in off-peak period and release the energy in peak period. To assess the recriticality potential for a liquid metal fast breeder reactor (LMFBR) following a core disruptive accident, El-Genk and Cronenberg [9] studied the freezing phenomena of molten fuel on a cold structure, and used the successive approximation technique to obtain a solution to the non-linear freezing problem. The effects or heat generation, viscous heat dissipation, temperature dependent thermophysical properties and a convective boundary condition at the solidification front have been incorporated into the an analytical formulation. Crepeau et al. [10] derived approximation governing equations of the interface location in one-dimensional PCM structure with internally generated heat in cylindrical, spherical, plane wall and semi-infinite geometries. Extending previous work, Crepeau et al. [11] investigated the solid–liquid phase change driven by volumetric energy generation in a vertical cylinder by a quasi-static, approximate analytical solution, and studied numerically the effect of convection within the liquid region. Crepeau and Siahpush [12] presented a quasi-static analytical solution of melting process in a cylinder with volumetric heat generation, which was valid for the Stefan number less than one. Kalaiselvam et al. [13] obtained analytical solutions of the interface locations at various time steps for PCM encapsulated inside cylindrical enclosures in solidification and melting process, and employed the model of conduction and heat generation for solidification problem and the model of conduction, convection and heat generation for melting problem to predict the transient interface positions and complete phase change time. Kalaiselvam et al. [14] presented the analytical and experimental investigation of the phase change heat transfer characteristics and thermodynamic behavior of spherically enclosed PCM with dispersion of nanoparticles for latent thermal energy storage (LTES) system in buildings. Assis et al. [15] investigated numerically and experimentally the process of melting of PCM in spherical geometry, and performed transient numerical simulations using the Fluent 6.0 software. Caldwell and Kwan [16] described and compared several effective methods for the numerical solution of one-dimensional Stefan problems, including melting in the half-plane, outward cylindrical solidification and outward spherical solidification.

Recently, Cotta and Mikhailov [17] proposed a systematic formalism to provide improved lumped parameter formulation for steady and transient heat conduction problems based on Hermite approximation for integrals that define averaged temperature and heat fluxes. This approach has been shown to be efficient in a variety of practical applications, such as nuclear fuel rods [18], spherical bodies [19], light water reactor (LWR) thermohydraulic analysis [20], ablative thermal protection for atmospheric reentry [21], single-layer slabs [22–24], multi-layer slabs [25] and adsorbed gas discharge operations [26]. The systematic formalism has not been widely applied in phase change heat conduction problems with or without internal heat generation, except Cotta et al. [27] and Mennig and Özişik [28].

The aim of this work is to develop a lumped parameter model for one-dimensional melting problems of a PCM slab with volumetric heat generation subjected to two types of boundary conditions respectively: (a) adiabatic condition at the left side and