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An empirical solution to turbulent natural convection and radiation heat transfer in square and rectangular enclosures

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HIGHLIGHTS

► Previous work has non-dimensionalised flow in enclosures with and without radiation.
► This extends the work by enabling a simple iterative technique to work out temperatures for total heat transfer rate.
► The provided solution has a maximum deviation of 7.7%.
► The method works for a variety of enclosures sizes, aspect ratios, temperatures and gases.

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ABSTRACT

The effects of natural turbulent convection with the interaction of surface radiation in a rectangular enclosure have previously been numerically and theoretically studied. The analyses were carried out over a wide range of enclosure aspect ratios ranging from 0.0625 to 16, different enclosure sizes, with cold wall temperatures ranging from 283 to 373 K, and temperature ratios ranging from 1.02 to 2.61. The work was carried out using four fluids (Argon, Air, Helium and Hydrogen; whose properties vary with temperature).

These can be used to calculate Nusselt number for pure natural convection and also to calculate the ratio between convection to radiation heat transfer for both square and rectangular enclosures.

This work extends these results by providing an empirical solution for the case of radiation and natural convection in square and rectangular enclosures and also provides a correlation equation to calculate the total Nusselt number for these cases. This method allows the simple calculation of either the total heat transfer rate, given the fluid, the geometry and the temperatures of the hot and cold walls, or via a straightforward iterative technique, the temperature of one wall given the other wall temperature and the total heat transfer.

1. Introduction

In a rectangular enclosure with natural convection and radiation as shown in Fig. 1, the internal flow is dominated by buoyancy forces. The most important dimensionless group in natural convection inside this enclosure is the Rayleigh number (which is the ratio of buoyancy forces to viscous forces acting on a fluid) which, for buoyancy dominated flows, is analogous to the Reynolds number. The value of the Rayleigh number can indicate whether the flow can be considered as laminar or turbulent [1,2].

In the studies of pure natural convection inside enclosed spaces, mostly two simple enclosed spaces are considered; first the square enclosure and secondly the rectangular cavity both with heated and cooled walls. The interest in this problem over the last four decades has led to many numerical and experimental studies. Elsherbiny et al. [3] reported experimentally the laminar natural convection across vertical and inclined air layers for different enclosure aspect ratios. They provided correlation equations to calculate Nusselt numbers. Davis [4] provided a bench-mark numerical solution for the natural convection of air in a square cavity and compared this to 37 other pieces of work. He [5] also provided a comparison exercise to confirm the accuracy of the bench mark solution in ref. [4]. Schmidt et al. [6] compared the experimental and predicted results for laminar natural convection in a water filled enclosure. Zhong et al. [7] studied the laminar natural convection in a square enclosure.

They observed that; with the effect of variable properties, when the temperature increases, the addition of this parameter slows

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Nomenclature

C_p	specific heat capacity (J kg^{-1} K^{-1})
g	gravitational acceleration (m s^{-2})
h	heat transfer coefficient (W m^{-2} K^{-1})
K	thermal conductivity of the fluid (W m^{-1} K^{-1})
L	enclosure wall length (m)
Q_{conv}	convection heat transfer (W)
Q_{rad}	radiation heat transfer (W)
Q_t	total heat transfer $= Q_{conv} + Q_{rad}$ (W)
T	temperature (K)
ρ	density (kg m^{-3})
μ	viscosity (N s m^{-2})
$\propto$	thermal diffusivity (m^2 s^{-1})
ϑ	kinematic viscosity (m^2 s^{-1})
β	thermal expansion coefficient (K^{-1})
ε	surface emissivity (−)
σ	Stefan–Boltzmann constant $= 5.672 \times 10^{-8}$ (W m^{-2} K^{-4})
ΔT	temperature difference between hot and cold walls (K)
Dimensionless groups	
Gr	Grashof number $= \beta g \Delta T L^3 \rho^2/\mu^2$ (−)
Nu_c	convection Nusselt number ($Q_{conv}/Q_{cond} = hL/K$) (−)
Nu_r	radiation equivalent Nusselt number (Q_{rad}/Q_{cond}) (−)
Nu_t	total Nusselt number ($Q_{conv} + Q_{rad}/Q_{cond}$) (−)
Pl	Planck number $= (Q_{cond}/Q_{rad})$ (−)
Pr	Prandtl number $= \mu C_p/K$(−)
RC_n	the new dimensionless group $= Nu_c/Nu_r = Q_{conv}/Q_{rad} \approx h/\sigma\varepsilon T^3$ (−)
Ra	Rayleigh number ($g\beta\Delta T L^3/\vartheta\propto$) (−)
T_r	temperature ratio $= T_h/T_c$ (−)

down the vertical velocities in the hot wall region and increases these velocities in the core but at the same time, the total heat transfer rate is seen to increase. Also, they pointed out the necessity of solving realistic physical cases (i.e. by including the thermal radiation effects with the variable properties). Fusegi and Hyun [8] reported the effects of complex and realistic conditions such as variable properties and three-dimensionality on laminar and transitional natural convection in an enclosure. They have thrown light on the discrepancies between the numerical prediction and the experimental measurements; to the unexplored aspects of realistic flows. Nithyadevi et al. [9] investigated the effect of aspect ratio on the natural convection in a rectangular cavity with partially active side walls. They found that, the heat transfer rate is increase with an increase in the aspect ratio.

The importance of surface radiation with natural convection in enclosures has been studied and investigated by many researchers. Velusamy et al. [10] studied the turbulent natural convection with the effect of surface radiation in square and rectangular enclosures. They pointed out that, the radiation heat transfer is significant even at low temperatures and low emissivities. Colomer et al. [11] looked at the three-dimensional numerical simulation of the interaction between the laminar natural convection and the radiation in a differentially heated cavity for both transparent and participating media. Their work reveals that in a transparent fluid, the radiation significantly increases the heat flux across the enclosure. Sen and Sarkar [12] have considered the effects of variable properties on the interaction of laminar natural convection and surface radiation in a differentially heated square cavity. They discovered that, the presence of both variable properties and radiation at low emissivity (ε = 0.1), affects the thermal stratification of the core and the symmetry of the mid-plane vertical velocity, as well as temperature profiles. Akiyama and Chong [13] analysed the interaction of laminar natural convection with surface thermal radiation in a square enclosure filled with air. They found that the presence of surface radiation significantly altered the temperature distribution and the flow patterns and affected the values of average convective and radiation Nusselt numbers.

Patterson and Imberger [14] studied the transient natural convection in a cavity of aspect ratios less than one. They provide a scaling analysis of the heat transfer inside a rectangular enclosure. Bejan et al. [15] experimentally studied the turbulent natural convection in shallow enclosures. Their experimental study focused on the flow and temperature patterns in the core region. Bejan [16] later explained the pure natural convection of heat transfer flow regimes in tall, square and shallow enclosures. He claims that the relationship between the Nusselt and Rayleigh numbers in

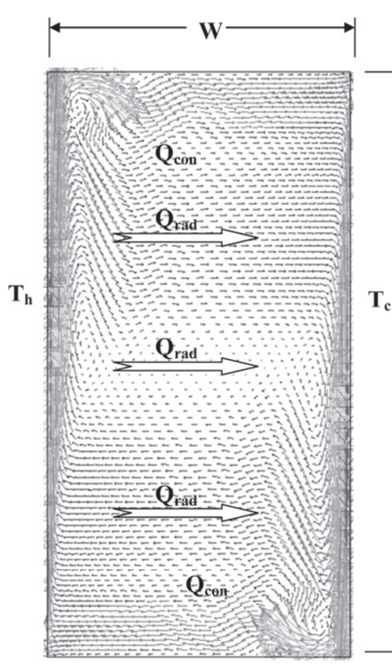

Fig. 1. Schematic diagram of the heat transfer inside a rectangular enclosure.

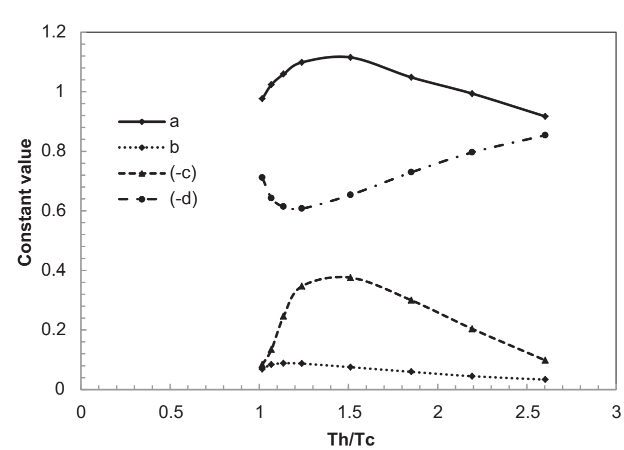

Fig. 2. Constants for equation (12) as a function of temperature ratio.