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On fixed-point iterations for the solution of control equations in power systems transients

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ABSTRACT

This paper contributes toward the establishment of a formal analysis method of control system equations solved through fixed-point iterations. The success of fixed-point iterations relies on contraction properties of the function to be iterated. A convergence criterion is presented and accuracy is not sacrificed over gain in computational performance.

The presented algorithms are illustrated in EMTP-RV for practical control systems used in wind power generation and for a user defined model case. Limitations and performances are discussed in relation to the Newton method.

1. Introduction

AN iterative Newton method for the solution of control system equations in electromagnetic transient (EMT) type simulation methods has been proposed in [1]. Although it represents a robust and systematic approach, there are some feedback based control systems that can be also solved using the much simpler and sometimes more efficient fixed-point (FP) method. The efficiency level of the FP method can be very high since it only sequentially evaluates the control blocks and does not require time-consuming linearization procedures and matrix formulations required in the Newton method. The difficulty is in the determination of whether or not the FP method can converge for a given case, before it is actually undertaken.

Moreover, in some classes of control system equations, the model loops may lead to algebraic constraints. In such cases, the basic sequential evaluation of blocks is not applicable. Different approaches can be undertaken to reformulate models in order to apply a sequential solution. The approach proposed in [2] consists of breaking algebraic loops. This approach is acceptable when the loop is artificial, *i.e.* when it can be eliminated without compromising the physical behavior of the model. Specific tools are dedicated to achieve such elimination [3]. However, some cases require algebraic constraints that cannot be easily eliminated. A possible solution consists in re-organizing blocks to eliminate algebraic loops while still maintaining functionality, but as pointed out in [3], this may become prohibitively difficult.

Loop-breaking is valid for some classes of control system equations, but it was proven that it may fail for others, for instance, when nonlinear (NL) blocks appear in the feedback path [1]. A highly accurate algorithm should handle algebraic loops by solving a set of NL control equations simultaneously while maintaining computational performance, but this is not usually the case and iterative solutions may require longer computing times.

It is proposed in this paper to analyze control system equations to formally display the contractive properties of loop paths. The success of the FP method relies on contraction properties of the iterated function [4–7]. It is proposed, in this paper, to study the iterated functions by analyzing the Jacobian matrix in association with the isolated variables representing the feedback loop path. Graph theory techniques are used for that purpose. The consideration of such properties may widen the usage of FP methods. When the convergence criterion is established, solution accuracy is not sacrificed over reduction in computing time. Additional iterations permit achieving convergence for a predefined tolerance.

This paper contributes to the establishment of a formal analysis method of control system equations which permits safe usage of the FP method.

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The analysis proposed in this paper is illustrated for the simulation of practical control systems in the study of power system transients. The test cases are for wind power generation and user defined model equations for an electrical machine.

2. Theoretical background

The literature on theoretical and fundamental aspects related to the FP method is abundant [4]. We will thus restrict our review to the solution of the systems of equations of the form:

$$\mathbf{e} = \boldsymbol{\varphi}(\mathbf{e}) \tag{1}$$

where $\boldsymbol{\varphi}$ is a vector function and $\mathbf{e}$ is the vector of unknowns. A solution $\hat{\mathbf{e}}$ of (1) is said to be a FP of $\boldsymbol{\varphi}$, since $\boldsymbol{\varphi}$ leaves $\hat{\mathbf{e}}$ invariant. The classical approach starts by setting an initial vector $\mathbf{e}^0$ and computing $\mathbf{e}^1 = \boldsymbol{\varphi}(\mathbf{e}^0)$ to continue iteratively (iteration counter is k) with successive evaluations $\mathbf{e}^{k+1} = \boldsymbol{\varphi}(\mathbf{e}^k)$ until convergence. The contraction mapping theorem gives a sufficient condition under which there is a fixed point $\hat{\mathbf{e}}$ of (1). For more details, see, among others [4–6]. Formally a vector-valued function $\boldsymbol{\varphi}$ is a contraction at a point $\hat{\mathbf{e}}$ if a constant σ exists, with $0 \le \sigma < 1$, in such a way that

$$\|\boldsymbol{\varphi}(\mathbf{e}) - \boldsymbol{\varphi}(\hat{\mathbf{e}})\| < \sigma\|\mathbf{e} - \hat{\mathbf{e}}\|, \tag{2}$$

for all $\mathbf{e}$ sufficiently close to $\hat{\mathbf{e}}$ and where $||.||$ is a specific norm to be defined. The Euclidean norm can be used for control system simulations in power systems. Also, some assumptions are made including at least, that all elements of $\boldsymbol{\varphi}$ are piecewise-continuous and, also, that the derivatives of the control block functions are well defined and are not infinite. Similar assumptions are made for the Newton-like methods [6,8].

The following condition of contraction [7] is used in this paper:

$$\|\boldsymbol{\varphi}'(\hat{\mathbf{e}})\|_{\text{spectral}} < 1 \tag{3}$$

where $\|\,.\,\|_{\text{spectral}}$ is the spectral (induced) norm and $\boldsymbol{\varphi}'$ is the Jacobian matrix of $\boldsymbol{\varphi}$. The induced spectral norm of a matrix $\mathbf{A}$ is the square root of the largest eigenvalue of the matrix resulting from $\mathbf{A}^T\mathbf{A}$.

The vector-valued $\boldsymbol{\varphi}$ is a mapping of vector-valued, possibly nonlinear functions, defined through algebraic equations representing a discrete dynamical system. EMT-type simulations are based on the discretization of control system blocks (using the trapezoidal rule, for example) and at a given time-step, all terms including history, inputs and outputs of models can be expressed as (1).

3. Fixed-point iterations: formulation and applications

3.1. Functions in feedback paths

Simultaneous systems of equations can be represented as feedback equations. Basically, a proper cutset is introduced on the graph of control equations in such a way that all cycles are eliminated. The set of variables pertaining to that cutset represent the feedback variables $\hat{\boldsymbol{\beta}}$: the cycles which were removed by cutting the feedback variables represented by the feedback paths on the graph of the control system. The all-zero eigenvalues condition for the adjacency matrix of the graph can be applied for testing the elimination of all cycles [9,10]. This approach provides vector-valued functions $\mathbf{G}$ and $\boldsymbol{\varphi}$ for formulating the objective function $\boldsymbol{\Phi}$ in the application of the Newton method. For a generic case:

$$\mathbf{e} = \boldsymbol{\varphi}(\mathbf{u}, \mathbf{y}) \tag{4}$$

$$\mathbf{y} = \mathbf{G}(\mathbf{e}) \tag{5}$$

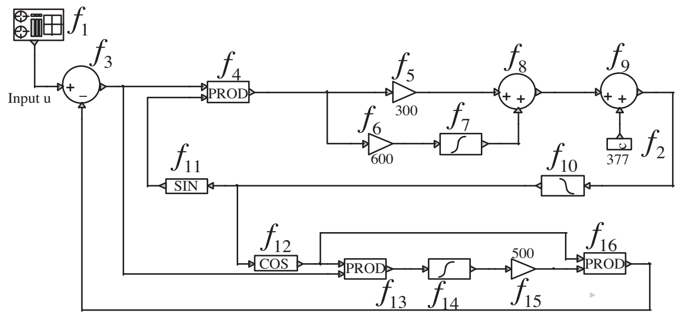

Fig. 1. Phase-locked Loop (PLL) control system from [1].

where $\mathbf{u}$ holds the vector of independent inputs, $\mathbf{y}$ is found using the application sequence $\mathbf{G}$ of the control diagram paths on $\mathbf{e}$ and the Newton solution is based on:

$$\boldsymbol{\Phi} = \mathbf{e} - \boldsymbol{\varphi}(\mathbf{u}, \mathbf{y}) = 0 \tag{6}$$

where $\mathbf{e}$ holds for feedback variables associated with the cutset $\hat{\boldsymbol{\beta}}$.

For illustration purposes, let us consider a Phase-Locked Loop (PLL) control studied in [1] and presented in Fig. 1. There is a set of two feedback functions $\{f_3, f_4\}$ in $\hat{\boldsymbol{\beta}}$. The equivalent system of equation is consequently given by:

$$\begin{aligned} e_1 &= u - G_1(e_1, e_2) \\ e_2 &= 0 + G_2(e_1, e_2) \end{aligned} \tag{7}$$

where the application sequences (sequential evaluations) are defined by:

$$\begin{aligned} G_1(\mathbf{e}) &= f_{16,15,14,13,12,10,9,8,7,6,5}(\mathbf{e}) \\ G_2(\mathbf{e}) &= f_{11,10,9,8,7,6,5}(\mathbf{e}) \end{aligned} \tag{8}$$

and $\mathbf{e} = [e_1, e_2]$ is the vector of new variables. The FP iterations are now defined as:

$$\mathbf{e}^{k+1} = \boldsymbol{\varphi}(\mathbf{G}(\mathbf{e}^k)) \tag{9}$$

Successive FP iterations on (9) will converge for some classes of control systems complying with the contraction mapping theorem recalled in section II. The convergence criterion adopted from [13] is given by:

$$\left|\frac{\|\boldsymbol{\Phi}^k\| - \|\boldsymbol{\Phi}^{k+1}\|}{\|\boldsymbol{\Phi}^k\|}\right| \le \Phi_{\text{tol}} \tag{10}$$

where Φ_{tol} is a relative tolerance on the objective function and $||.||$ is the Euclidian norm. The objective function (6) is rewritten as:

$$\boldsymbol{\Phi}^{k+1} = \mathbf{e}^{k+1} - \boldsymbol{\varphi}(\mathbf{G}(\mathbf{e}^k)) \tag{11}$$

3.2. Application cases

The following simulations have been performed using the EMTP-RV [14] engine with the added functionalities described in this paper.

3.2.1. Phase-Locked Loop

The method proposed in the previous section is applied to the PLL presented in Fig. 1. The contraction condition is respected for a sufficiently small time-step ($\Delta t = 1\,\mu$s) and the found solution is identical to the one from the Newton method using the same Δt. For this particular test with a sine input having a jump in amplitude value at 125 ms, when the time-step increases, the FP method performs poorly especially near the jump. In fact, numerical derivatives of nonlinear functions of blocks (PROD, sine and cosine) and the discretization by the trapezoidal rule (applied to