Electric Power Systems Research 97 (2013) 51–60

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A mixed-integer LP model for the reconfiguration of radial electric distribution systems considering distributed generation

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ARTICLE INFO

Article history:
Received 19 July 2012
Received in revised form
30 September 2012
Accepted 10 December 2012
Available online 16 January 2013

Keywords:
Reconfiguration of electric distribution systems
Active power losses reduction
Distributed generation
Mixed-integer linear programming

ABSTRACT

The problem of reconfiguration of distribution systems considering the presence of distributed generation is modeled as a mixed-integer linear programming (MILP) problem in this paper. The demands of the electric distribution system are modeled through linear approximations in terms of real and imaginary parts of the voltage, taking into account typical operating conditions of the electric distribution system. The use of an MILP formulation has the following benefits: (a) a robust mathematical model that is equivalent to the mixed-integer non-linear programming model; (b) an efficient computational behavior with exiting MILP solvers; and (c) guarantees convergence to optimality using classical optimization techniques. Results from one test system and two real systems show the excellent performance of the proposed methodology compared with conventional methods.

1. Introduction

The reconfiguration of the distribution system (RDS) is one of the classic optimization problems in the operation of electrical distribution systems (EDS). In addressing this problem, the main objective is to find the best radial topology in order to obtain minimum active power losses, meet the energy demand, and maintain system reliability. This procedure can permit an efficient and reliable operation. Due to various technical reasons, the EDS must operate with a radial topology (even though it has a mesh structure), the two most important factors being: (a) facilitation coordination and protection; and (b) achieving a reduction of short-circuit current in EDS. The RDS is a problem related to the operation planning of EDS and can be modeled as a highly complex mixed-integer nonlinear programming (MINLP) problem [1].

Some works presented in the specialized literature propose mixed-integer linear and quadratic programming models for the RDS problem. In [2], the authors present a linear model for the RDS problem using the so-called transportation method and a comparison with heuristics methods is made. In [3], the RDS problem is formulated as a minimum cost network flow problem and is solved using a modified simplex method, ignoring the branch capacity limits. In [4], the authors extend the method in [3] with the presence of distributed generation. In [5], two methodologies are used to solve the RDS problem. The first methodology models the RDS problem as a mixed-integer linear programming problem (due to the linearization of the objective function and constraints) and solves it using a standard optimization package. In the second technique, the same problem is modeled as a mixed-integer nonlinear problem and is solved by means of a genetic algorithm. The paper concludes that the results of both methodologies are similar for the tests completed. An extension from [5] is presented in [6], in which the RDS problem is modeled as a mixed-integer quadratic programming problem. A mixed-integer conic programming formulation for the RDS problem is shown in [7].

Several optimization techniques have been offered in the literature to solve the RDS problem. These techniques can be separated into two major groups: (1) exact techniques and (2) heuristic and metaheuristic techniques. The exact techniques, such as branch and bound algorithms, were used only for relaxed models [1,5,6]. However, more recently, heuristics and metaheuristics have been successfully applied with complete models. Simulated annealing [8,9], ant colony [10,11], particle swarm optimization [12], genetic algorithm [13–15] and tabu search algorithms (TSA) [16,17] are among the metaheuristic techniques used to solve the RDS problem. Constructive heuristic algorithms, as seen in [18–22], are among the heuristic algorithms used to solve the RDS problem.

In this paper, the problem of reconfiguration of electric distribution systems considering the presence of distributed generation is modeled as a mixed-integer linear programming (MILP) problem. Linearizations were made to represent adequately the steady-state operation of the EDS considering the behavior of the constant

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0378-7796/$ – see front matter.
http://dx.doi.org/10.1016/j.epsr.2012.12.005

power type load. The integer nature of the decision variables represents the state of the branches that can be opened or closed in the EDS. The objective is to minimize the active power losses subject to operational and physical constraints. The proposed model was tested using one test system of 69 nodes and two real systems of 136 and 417 nodes. In order to validate the approximations performed, a steady-state operation point was compared with that obtained using the single phase load flow sweep method.

The main contributions of this paper are as follows:

1. A novel model for the steady-state operation of a EDS through the use of linear expressions.
2. A novel MINLP model for the RDS problem where the radiality constraints are properly considered in the mathematical model. Additionally, an extension of this model considering the distributed generation (DG) operation is presented.
3. A MILP formulation for the RDS problem considering DG has the following benefits: (a) a robust mathematical model that is equivalent to the MINLP model; (b) an efficient computational behavior with exiting MILP solvers; and (c) guarantees convergence to optimality using classical optimization techniques.

2. A mixed-integer nonlinear model for the problem of reconfiguration of EDS

In order to model the problem of reconfiguration of EDS, the following assumptions are made:

1. The loads of EDS are modeled as constant power, constant current and constant impedance types.
2. The steady-state operation of EDS is represented in terms of the real and imaginary parts of the voltage and current flow.
3. The three phase EDS is considered symmetrical and then modeled through their positive sequence network.

The RDS problem can be modeled as a mixed-integer nonlinear programming problem as shown in (1)–(17).

$$\min v = \sum_{ij \in \Omega_l} \left[\left(I_{ij}^{re+} + I_{ij}^{re-} \right)^2 + \left(I_{ij}^{im+} + I_{ij}^{im-} \right)^2 \right] R_{ij} \tag{1}$$

subject to

$$\sum_{ki \in \Omega_l} \left(I_{ki}^{re+} - I_{ki}^{re-} \right) - \sum_{ij \in \Omega_l} \left(I_{ij}^{re+} - I_{ij}^{re-} \right) + I_{Gi}^{re} = I_{Di}^{re} \quad \forall i \in \Omega_b \tag{2}$$

$$\sum_{ki \in \Omega_l} \left(I_{ki}^{im+} - I_{ki}^{im-} \right) - \sum_{ij \in \Omega_l} \left(I_{ij}^{im+} - I_{ij}^{im-} \right) + I_{Gi}^{im} = I_{Di}^{im} \quad \forall i \in \Omega_b \tag{3}$$

$$V_i^{re} - V_j^{re} + w_{ij}^{re} = R_{ij}(I_{ij}^{re+} - I_{ij}^{re-}) - X_{ij}(I_{ij}^{im+} - I_{ij}^{im-}) \quad \forall ij \in \Omega_l \tag{4}$$

$$V_i^{im} - V_j^{im} + w_{ij}^{im} = X_{ij}(I_{ij}^{re+} - I_{ij}^{re-}) + R_{ij}(I_{ij}^{im+} - I_{ij}^{im-}) \quad \forall ij \in \Omega_l \tag{5}$$

$$I_{Di}^{re} = \frac{P_{Di} V_i^{re} + Q_{Di} V_i^{im}}{V_i^{re2} + V_i^{im2}} \quad \forall i \in \Omega_b \tag{6}$$

$$I_{Di}^{im} = \frac{P_{Di} V_i^{im} - Q_{Di} V_i^{re}}{V_i^{re2} + V_i^{im2}} \quad \forall i \in \Omega_b \tag{7}$$

$$(I_{ij}^{re+} + I_{ij}^{re-})^2 + (I_{ij}^{im+} + I_{ij}^{im-})^2 \le \bar{I}_{ij}^2 (y_{ij}^+ + y_{ij}^-) \quad \forall ij \in \Omega_l \tag{8}$$

$$\underline{V}^2 \le V_i^{re2} + V_i^{im2} \le \overline{V}^2 \quad \forall i \in \Omega_b \tag{9}$$

$$|w_{ij}^{re}| \le \overline{w}_{ij}^{re}(1 - y_{ij}^+ - y_{ij}^-) \quad \forall ij \in \Omega_l \tag{10}$$

$$|w_{ij}^{im}| \le \overline{w}_{ij}^{im}(1 - y_{ij}^+ - y_{ij}^-) \quad \forall ij \in \Omega_l \tag{11}$$

$$0 \le I_{ij}^{re+} \le \bar{I}_{ij} y_{ij}^+ \quad \forall ij \in \Omega_l \tag{12}$$

$$0 \le I_{ij}^{re-} \le \bar{I}_{ij} y_{ij}^- \quad \forall ij \in \Omega_l \tag{13}$$

$$y_{ij}^+ + y_{ij}^- \le 1 \quad \forall ij \in \Omega_l \tag{14}$$

$$\sum_{ij \in \Omega_l} (y_{ij}^+ + y_{ij}^-) = |\Omega_b| - 1 \tag{15}$$

$$I_{ij}^{im+}, I_{ij}^{im-} \ge 0 \quad \forall ij \in \Omega_l \tag{16}$$

$$y_{ij}^+, y_{ij}^- \in \{0, 1\} \quad \forall ij \in \Omega_l \tag{17}$$

The objective function of the RDS problem is the minimization of the active power losses of an EDS, as shown in (1). Note that the real part of the current flow of branch ij is represented by two positive variables I_{ij}^{re+} and I_{ij}^{re-}, according to the direction of the current flow; similarly, variables I_{ij}^{im+} and I_{ij}^{im-} are used for the imaginary part of the current flow for branch ij. Constraints (2) and (3) represent respectively the balance of real and imaginary parts of the nodal current at node i (see Fig. 1). Constraints (4) and (5) represent, respectively, the real and imaginary parts of the voltage drop in branch ij. The real and imaginary parts of the currents demanded by the loads are determined by (6) and (7), where P_{Di} and Q_{Di} varies according to the voltage magnitude at node i, as shown in Appendix A, to represent the loads as constant power, constant current and constant impedance loads. Constraints (8) and (9) represent the current flow capacity of each branch and the limits of the voltage magnitude, respectively.

The state of the branch ij is determined by the binary decision variables y_{ij}^+ and y_{ij}^-. If y_{ij}^+ or y_{ij}^- are equal 1, then the branch ij is in operation and if both y_{ij}^+ and y_{ij}^- are zero then the branch ij is out of operation. Despite the fact that the state of a circuit can be represented using only a binary variable, the use of two binary variables makes it possible to limit the direction of the real part of the current flow in the circuit (one binary variable is associated with the forward direction, while the other is associated with the backward direction), which yields better performance. Constraints (10) and (11) state that auxiliary variables w_{ij}^{re} and w_{ij}^{im} are zero if branch ij is in operation. Constants $\overline{w}_{ij}^{re}$ and $\overline{w}_{ij}^{im}$ must be calculated to give a sufficient degree of freedom for w_{ij}^{re} and w_{ij}^{im} in order to satisfy (4) and (5) where branch ij is out of operation. Constraints (12) and (13) define the direction of the real part of the current flows in function of the binary variables y_{ij}^+ and y_{ij}^-, respectively; and constraint (14) ensures that duplication of current flow directions (forward and backward) is not allowed. Note that if $y_{ij}^+ = 1$, then $y_{ij}^- = 0$, $I_{ij}^{re-} = 0$, I_{ij}^{re+} is nonzero and the current flow direction is forward. If $y_{ij}^- = 1$, then $y_{ij}^+ = 0$, $I_{ij}^{re+} = 0$, I_{ij}^{re-} is nonzero and the current flow direction is backward. Note that $\bar{I}_{ij}$ in (12) and (13) represents the degree of freedom of the variables I_{ij}^{re+} and I_{ij}^{re-} when $y_{ij}^+ = 1$ or $y_{ij}^- = 1$ respectively.

Constraint (15), combined with (2) and (3), is used to obtain a radial topology for the RDS problem, as shown in [23]. The condition of non-negativity for the variables I_{ij}^{im+} and I_{ij}^{im-} is stated in (16). The binary nature of the decision variables is represented by (17), and a feasible operation solution for the EDS depends on their value. The remaining variables represent the operating state of a feasible solution. For a feasible investment proposal, defined through the specified values of y_{ij}^+ and y_{ij}^-, several feasible operation states are possible. Given that R_{ij} is positive in value, the objective function (1) is a convex quadratic function. Constraints (2)–(5) and (10)–(16) are linear, while (6)–(9) are non-linear. With the aim of using a commercial solver, it is desirable to obtain a linear equivalent for (6)–(9) and for the objective function (1).