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Boundary integral representations of steady flow around a ship

Jiayi He, Yi Zhu, Chao Ma, Chen-Jun Yang, Wei Li, Francis Noblesse *

State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean & Civil Engineering and Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai Jiao Tong University, Shanghai, China

HIGHLIGHTS

- The classical NK theory of steady flow around a ship hull and the related NM theory are revisited and significantly expanded.
- A new basic boundary-integral identity is given, and five related alternative boundary-integral representations are considered.

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ABSTRACT

The classical Neumann–Kelvin (NK) linear potential flow theory of ship waves in calm water and the related Neumann–Michell (NM) theory are considered. Five alternative boundary integral representations are given: (1) the classical NK integro-differential representation, called "classical NK formulation", which corresponds to an inconsistent linear flow model, (2) a modification of the classical NK flow formulation that corresponds to a consistent linear flow model and is called "consistent NK formulation", (3) a flow representation, called "NM potential and velocity formulation", that involves the flow potential ϕ and the velocity components ϕ_d and ϕ_t along two unit vectors **d** and **t** tangent to the ship hull surface, and yields an integro-differential equation for determining ϕ, (4) a flow representation, called "NM velocity formulation", that only involves ϕ_d and ϕ_t and yields a pair of coupled integral equations for determining (ϕ_d, ϕ_t), and (5) a flow representation called "NM potential formulation" that only involves the flow potential ϕ and yields an integral equation for determining ϕ. The two NK formulations involve both a surface integral over the ship hull surface and a line integral around the ship waterline, whereas the three NM formulations do not involve a waterline integral. All flow representations other than the classical NK representation are based on a consistent linear flow model.

1. Introduction

The flow around a ship of length L that travels at a constant speed V along a straight path, in calm water of large depth and horizontal extent, is considered. The wave drag related to the waves generated by the ship hull is of considerable practical importance because drag is a critical and dominant hydrodynamic element of ship design. Accordingly, prediction of ship waves is a classical fluid flow problem that has been widely considered in a huge body of literature. Indeed, alternative methods have been developed and can be used to compute steady free-surface flows around ship hulls. A partial review of this literature may be found in e.g. [1], and a number of studies that are related to the analysis considered here are listed further on. Routine applications to ship design, especially early-stage design and hull-form optimization, necessitate robust and practical flow computation methods, and yet require realistic theories that account for the dominant flow physics and are then sufficiently accurate.

The Neumann–Michell (NM) theory expounded in [1] is a linear potential flow theory that is a modification of the classical Neumann–Kelvin (NK) theory. Unlike the NK theory, the NM theory corresponds to a consistent linear flow model, and moreover does not involve a line integral around the mean ship waterline. An important feature of the NM theory is that it is a very practical theory. Indeed, the flow around a ship hull can be evaluated in about 1 s via a PC. The theory is then well suited for routine applications to ship design and hydrodynamic optimization, and in fact has already been widely used for hull-form optimization; e.g. [2–10].

The NM theory is shown in [11–18] to yield realistic predictions of the drag, the sinkage and the trim experienced by a ship, as well as the wave profile along a ship hull, that are in satisfactory overall agreement with experimental measurements and are sufficiently accurate for practical purposes, notably for early design and hull-form optimization, within a broad range of Froude numbers. In

* Corresponding author.
E-mail address: noblfranc@gmail.com (F. Noblesse).

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particular, [16,17] show that the influence of sinkage and trim on the drag of a freely-floating monohull ship can readily be taken into account in the NM theory, and [18] presents simple post-processing nonlinear corrections (without additional flow computations) of the NM linear theory that account for dominant nonlinear effects (notably the substantial decrease in the wave drag that occurs for a ship with a large bulbous bow). Furthermore, flow predictions obtained via the NM theory or via far more complicated CFD methods compare well, as is illustrated further on in this study.

The NM theory is revisited and significantly expanded here. In particular, a new basic boundary-integral identity, and two new boundary-integral formulations of the NM theory, are given. These two alternative formulations of the NM theory only involve the flow velocity or the flow potential and are then called "NM velocity formulation" or "NM potential formulation", and supplement the "NM potential and velocity formulation" previously given in [1], which involves both the flow potential and the flow velocity.

The Froude number F is defined as

$$F \equiv V/\sqrt{gL} \tag{1}$$

where g denotes the acceleration of gravity. The flow due to the ship is observed in a system of orthogonal coordinates $\mathbf{X} \equiv (X, Y, Z)$ attached to the moving ship. The undisturbed free surface, denoted as Σ_F, is chosen as the plane $Z = 0$ with the Z axis directed upward. The X axis is taken along the ship path and points toward the ship bow. The flow thus appears steady with flow velocity given by the sum of the apparent uniform current $(-V, 0, 0)$ that opposes the ship speed V and the (disturbance) flow velocity given by the gradient (Φ_X, Φ_Y, Φ_Z) of the flow potential $\Phi(\mathbf{X})$. The length L and the speed V of the ship are used to define the nondimensional coordinates $\mathbf{x} \equiv \mathbf{X}/L$, flow velocity $(\phi_x, \phi_y, \phi_z) \equiv (\Phi_X, \Phi_Y, \Phi_Z)/V$ and flow potential $\phi \equiv \Phi/(VL)$.

The mean wetted hull surface of the ship is denoted as Σ_H. This surface intersects the undisturbed free-surface plane $z = 0$ along the mean ship waterline Γ, which is oriented in the clockwise direction when viewed from above the free surface. The unit vector $\mathbf{n} \equiv (n^x, n^y, n^z)$ is normal to the hull surface Σ_H and points outside the ship. D denotes the mean flow region bounded by the hull surface Σ_H and the free surface Σ_F.

2. Linear flow models and Neumann–Kelvin theory

The theoretical basis of the Neumann–Kelvin (NK) theory can be questioned on the grounds that the NK linear flow model does not correspond to a clear "cause-to-effect" linearization scheme, unlike the classical linear flow models for a body submerged at a sufficiently large depth or for a free-surface-piercing ship hull that is sufficiently thin, flat, or slender. Indeed, thin-ship, flat-ship, and slender-ship theories, as well as free-surface flows around fully-submerged bodies, have been widely considered in the literature; e.g. [19–32]. However, the assumptions that a body is deeply submerged or that a ship hull is very thin, flat or slender are sufficient, but not necessary, linearization assumptions.

Typical ships are 3D streamlined slender bodies that only create relatively small flow disturbances at the free surface. The assumption that the boundary condition at the free surface can be linearized is then a reasonable approximation. Indeed, [18] shows that free-surface nonlinearities for common displacement ships are weak and have a limited influence on the wave profile, except at a ship bow where nonlinear effects can be very large.

In particular, [33,34] show that two main ship bow wave reg imes, the "overturning bow wave regime" and the "unsteady bow wave regime", exist. A ship bow wave in the overturning bow wave regime consists of an overturning thin sheet of water that is mostly steady until it reenters the free surface. In the unsteady bow wave regime, a ship (that advances at a constant speed in calm water) creates a highly turbulent unsteady bow wave (rather than a steady bow wave). [33,34] also show that the overturning bow wave regime and the unsteady bow wave regime mostly occur for fast ships that have fine bows or for slow ships that have blunt bows, respectively.

A fully nonlinear theory of nearfield flow around a ship hull is then extremely difficult, and it arguably is best to ignore nonlinearities associated with the boundary condition at the free surface in a practical theory meant for routine applications to ship design and hull-form optimization.

The boundary condition at the free surface in the classical thin-ship, flat-ship, slender-ship and NK theories is linearized about the uniform stream $(-V, 0, 0)$. This classical linear free-surface boundary condition, widely adopted since Kelvin [35] and Michell [19], assumes that the waves created by a ship are small perturbations of the uniform stream $(-V, 0, 0)$.

Linearization of the free-surface boundary condition about the flow around a ship hull and its mirror image with respect to the plane $z = 0$ of the undisturbed free surface, commonly called "double-body" flow in the literature, has also been used; e.g. [36–41]. This alternative linearization assumes that the waves created by a ship are small perturbations of the double-body flow. The double-body flow, identical to the flow around the ship hull in the limit $F = 0$ in which the free surface is a rigid wall, does not significantly differ from the uniform stream $(-V, 0, 0)$ for common slender ships except near the point where the ship bow intersects the plane $z = 0$, which is a stagnation point of the double-body flow.

Linearization of the free-surface boundary condition about the double-body flow may be realistic for slow ships with blunt bows. However, this linearization is ill suited for ships with fine bows because the double-body flow varies very rapidly in the vicinity of the point where a wedge-like ship bow intersects the plane $z = 0$, and indeed is singular at the intersection point in the limit when the entrance angle of the ship bow vanishes. Accurate numerical evaluation of the derivatives of the double-body flow potential – required in the free-surface boundary condition associated with linearization about the double-body flow – is then difficult for fine ship bows, and significant numerical inaccuracies can occur in this common case.

Moreover, the nonlinear analysis given in [42] of the free-surface flow in the vicinity of the point where a ship bow intersects the free surface shows that the local free-surface flow at a ship bow greatly differs from the stagnation flow predicted by the double-body flow model at this point. In fact, [42] shows that the nonlinear local free-surface flow in the vicinity of a ship bow is not a small perturbation about either the uniform stream $(-V, 0, 0)$ or the double-body flow, as is assumed in the Kelvin–Michell or the double-body flow linearizations. Linearization about the double-body flow, which is nearly identical to the uniform flow $(-V, 0, 0)$ except in the vicinity of the point where a ship bow intersects the free surface as was already noted, is then no better justified than the Kelvin–Michell linearization, which has the huge merit of being based on a uniform flow unaffected by numerical inaccuracies. The Kelvin–Michell linearization is adopted in the Neumann–Kelvin theory considered here.

3. Neumann–Kelvin boundary-value problem

The velocity potential

$$\phi(\mathbf{x})\, e^{\mu T} \quad \text{where } 0 < \mu \ll 1$$

is associated with a flow that slowly grows from rest at time $t \equiv TV/L = -\infty$. The NK theory considers the boundary-value problem defined by the Laplace equation

$$\nabla^2\phi = 0 \text{ in the mean flow region } D, \tag{2a}$$