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Secondary resonance of liquid sloshing in square-base tanks undergoing the circular orbit motion

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HIGHLIGHTS

- Second-order resonance in 3D sloshing tank undergoing orbit motion is studied.
- Resonant swirling waves with dominant even sloshing mode are observed.
- Three wave patterns during second-order resonance are identified.
- Mode-to-mode interaction due to nonlinear effects is found during second-order resonance.

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ABSTRACT

The transient wave sloshing in the square-base tank horizontally shaken in a circular orbit is numerically studied. The liquid sloshing is simulated by the boundary element method (BEM) based on the fully-nonlinear potential-flow theory. The tank is firstly excited at the first odd natural sloshing frequency. Resonant swirling waves are observed travelling along the tank sides, when the even sloshing mode is aroused. Then, the tank is excited at half of the first even natural frequency. Techniques of the FFT filter and wavelet analysis are applied to distinct the wave components from the wave elevation histories, through which the occurrence of the secondary resonance is identified. During the secondary resonance, three typical wave motion patterns are observed, i.e. swirling waves, standing waves and double-peak travelling waves. Effects of the excitation amplitude and the liquid depth on the secondary resonance are investigated. Further, the secondary resonance by oscillating the tank at the difference of the first even and first odd natural frequency is studied. The first odd sloshing mode is found to contribute to the dominance of the even mode wave component during the secondary resonance.

1. Introduction

The liquid sloshing can be found in many engineering applications, such as the fuel storages of aerospace crafts, large dams in the earthquake area, oil storage tanks, and cargo tanks of the liquefied natural gas (LNG) carrier [1–6]. Under certain excitation conditions, the liquid motion can grow resonant with large amplitudes, and endanger the structures and facilities in the tank. Thus, the sloshing resonance greatly concerns the safety of the above applications. However, the sloshing at resonance is featured by complex nonlinearity, and the corresponding nonlinear mechanism has become an important direction of the recent studies.

From the experiments, Faltinsen et al. [7,8] found that, when a liquid tank horizontally oscillates at a frequency close to the first odd natural frequency, the combination of the exciting and natural frequencies due to nonlinearities can lead to an amplification of higher-order natural sloshing modes. Here, the natural sloshing modes are non-trivial solutions of the tank sloshing. This was also indicated later in Hermann and Timokha [9], even although the sloshing modes are independent of one another from the linear wave theory. Further, from a mathematical derivation up to the second-order nonlinearity, Wu [10] predicted that, even if the excitation frequency was away from all the natural frequencies, higher-order sloshing modes can still become dominant due to the interaction of wave components. This leads to a type of resonance called the 'secondary resonance'.

After that, numerical studies on the secondary resonance can be found in a few publications. Firouz-Abadi et al. [11] developed a modal approach with the help of the boundary element method (BEM). The secondary resonance phenomena were observed in 2D wall-sided tanks with various bottoms. However, the simulation was still based on the second-order free-surface boundary conditions in a weakly nonlinear sense. Then, Ning et al. [12] applied the fully-nonlinear free-surface conditions and simulated the sloshing in 2D tanks using the higher-order BEM. A secondary resonance

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condition was investigated when the tank was horizontally excited at half of the first even natural frequency. The secondary resonance characterized by the dominant even sloshing mode was observed.

In the above 2D studies, the liquid tank is forced to oscillate in the tank length direction, and planar waves are generated as a result. However, for the resonant sloshing, complex 3D nonlinear effects are also important [7,8,13]. The secondary resonance in 3D sloshing tanks was subsequently carried out in some recent studies. Chern et al. [14] developed a pseudo-spectral matrix-element (PSME) method to investigate the secondary resonance in square-base tanks. The secondary resonance condition was satisfied mainly in the tank length direction, and the secondary resonance was still in a two-dimensional sense. Further, Zhang et al. [15] followed the perturbation analysis method in Wu [10] and derived the 3D secondary resonance conditions for square-base tanks. With these analytical predictions, Zhang et al. [16] conducted a series of numerical studies on the secondary resonance in square-base tanks. A parallelled 3D BEM based on the fully-nonlinear potential-flow theory was developed. The occurrence of the secondary resonance was identified through a spectral analysis of the free-surface elevation time histories at different stages. It showed that the secondary resonance can be triggered due to the interaction of two orthogonal plane waves.

Following Zhang et al. [16], this study further considers the secondary resonance of swirling waves in square-base tanks. The idea is enlightened by Wu et al. [17], Reclari et al. [18] and Reclari [19]. Wu et al. [17] observed in their numerical results that some new spectral peaks emerged at certain even natural sloshing frequencies or twice the excitation frequency, every time sloshing waves were doing the 'swirling' motion in a square-base tank. They attributed these phenomena to the higher-order resonance. Reclari et al. [18] conducted an extensive experimental survey on swirling waves in the circular cylindrical tank shaken in an orbit. It was observed that the wave crest along the tank wall may have multiple sub-crests at certain excitation frequencies, indicating a higher-order nonlinear wave phenomenon. More experimental data on the nonlinear wave phenomena in a circular cylindrical tank subject to orbital shaking were further given in Reclari [19]. A large variety of free surface shapes was identified, ranging from single and multiple crested waves to breaking waves and waves having a shape constantly changing as they rotate. The importance of free surface natural modes and their sub-harmonics in the behaviour of the waves was identified. The above literature suggests that the higher-order sloshing nonlinearity may be more evident for swirling waves. This may be understood as follows. For swirling waves, different wave components with different speeds are continuously chasing with each other along the tank walls from time to time. After a sufficient time of accumulation, the nonlinear effects due to the wave–wave interaction may become dominant. From this deduction, this study presumes that the secondary resonance may also become evident if the liquid swirls in the tank.

For a membrane-type LNG carrier in practice, the carrier can undergo a six-degree-of-freedom movement in ocean waves. Swirling waves can be easily excited in its square-base tanks. Identification of nonlinear properties of swirling waves in these tanks can help figure out the critical conditions that endanger the operation safety. Therefore, in this study, swirling waves in square-base tanks are investigated. In this study, the swirling motion of the liquid is generated by shaking the tank in the circular orbit. In the context of the membrane-type LNG tank, the fluid viscous effects are limited to a boundary layer close to the tank surface, outside which the flow can be considered to be frictionless and irrotational. Since the sloshing flow is at a high Reynolds number, the boundary layer's thickness becomes very small compared to the dimensions of the tank, so that the presence of the boundary layer is negligible and the sloshing domain could also be regarded as the potential-flow region. The 3D BEM developed in Zhang et al. [16] will be used here to investigate the secondary resonance of liquid sloshing. Section 2 will give the mathematical equations for the sloshing problem, and Section 3 will discuss numerical results and observations. Conclusions will be drawn in Section 4.

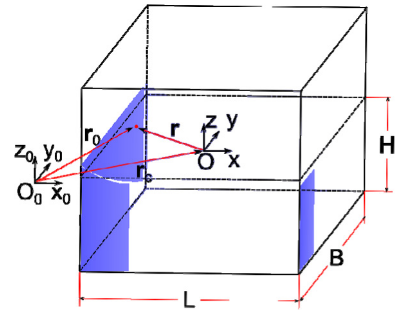

Fig. 1. Definition of coordinate systems.

2. Mathematical formulations

The square-base tank with length L and width B is considered. Two right-handed Cartesian coordinate systems, the earth-fixed system $O_o - x_o y_o z_o$ and the tank-fixed system $O - xyz$, are defined to describe the sloshing problem, as shown in Fig. 1. These two systems initially coincide with each other with the origin at the centre of the initial free surface. The Ox axis is set parallel to the length direction of the tank, and the Oz axis points vertically upward.

The potential-flow theory is used to describe the sloshing problem, assuming the fluid to be inviscid, incompressible and flow-irrotational. The velocity potential $\varphi(x, y, z, t)$ whose gradient is the fluid velocity is introduced, which is determined by the following boundary value problem (BVP):

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0, \text{ in the fluid domain } V \tag{1}$$

$$\frac{\partial\varphi}{\partial n} = \mathbf{v}_c \cdot \mathbf{n}, \text{ on the wet tank surface } S_B \tag{2}$$

$$\frac{\partial\eta}{\partial t} = -\mathbf{v}_c \cdot \left\{-\frac{\partial\eta}{\partial x}, -\frac{\partial\eta}{\partial y}, 1\right\} - \frac{\partial\varphi}{\partial x}\cdot\frac{\partial\eta}{\partial x} - \frac{\partial\varphi}{\partial y}\cdot\frac{\partial\eta}{\partial y} + \frac{\partial\varphi}{\partial z}, \text{ on the free surface } S_F \tag{3}$$

$$\frac{\partial\varphi}{\partial t} = -\frac{1}{2}\nabla\varphi\cdot\nabla\varphi - g\eta + \mathbf{v}_c\cdot\nabla\varphi, \text{ on the free surface } S_F \tag{4}$$

where $\mathbf{n}$ is the unit normal vector pointing out of the fluid domain, $\eta(x, y, t)$ denotes the free-surface elevation, g is the gravitational acceleration, and $\mathbf{v}_c = \{dx_c/dt, dy_c/dt, dz_c/dt\}$ is the velocity of the tank. In this study, the displacement of the tank is defined as

$$x_c = A\cos(\omega t); \quad y_c = A\sin(\omega t); \quad z_c = 0. \tag{5}$$

For completeness, the initial conditions are set as

$$\varphi(x, y, z, t) = 0; \quad \eta(x, y, t) = 0 \quad \text{for} \quad t \le 0 \tag{6}$$

which assumes that the fluid starts its motion from stationary.

The BEM is used to solve the BVP with respect to the velocity potential. Firstly, the governing Laplace's equation is transformed into the boundary integral equation (BIE) based on the Green's third identity. Then, the fluid boundary is discretized into the