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A study on supercavitation in axisymmetric subsonic liquid flow past slender conical body

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ABSTRACT

A novel theoretical model for solving the cavities in axisymmetric supercavitating flow past a slender conical body in subsonic fluid has been established in the present paper based on the slender body theory. The fluid compressibility has been taken into consideration in the present model. The nonlinear integral differential equation is derived for solving subsonic supercavitating flow. The numerical discrete method and the iterative process to solve the equation are presented in this paper. The critical Mach number are obtained to describe the subsonic flow. The results of supercavity shapes and the hydrodynamic coefficients obtained by the present theoretical model are compared with the results of other literatures, which verifies the present model have theoretical accuracy and broad application. Finally we discuss the compressibility effects on cavity shape, surface pressure distribution and drag coefficient in the subsonic liquid flow.

1. Introduction

Supercavitating vehicles can achieve high-speed in comparison with conventional vehicles for the reason that supercavitation can reduce the friction drag significantly. Much effort has been made in the past decades to study supercavitation around a moving underwater vehicle. The high-speed supercavitating projected has been focused attention on. Comprehensive reviews of this subject have been given in the literature [1–4]. These reviews outline the significant progress in the development of theoretical research and experimental research on high-speed supercavitating.

In the past, many experiments have been conducted to study supercavitation in high-speed flow. Yu. Yakimov [5] has carried out some experiments whose speeds were up to 1000 m/s. High speed experiments in the speed range of 500 m/s–1400 m/s were performed by Vlasenko [6]. Vlasenko's experiments results show that the cavity aspect ratio which is the ratio of half cavity length to cavity middle section diameter is very large. The cavity aspect ratio for speed range of 500 m/s–1400 m/s is 70–200 while the cavity aspect ratio is 10–20 when the cavitation number is larger than 0.01. The water compressibility is shown in photographs near the cavitator with the phenomenon that the local water density is changed which results in the distortion of the scale grid lines due to the water compressibility effect. Kirschner [7] stated the experiment program where the underwater Mach number can exceed unit and experiments speeds up to 1549 m/s with the Mach number 1.03 were achieved. These high-speed under water projectile experiments showed that the projectile travels along nearly straight line and the cavitation number increases and the cavity size decreases with time as a result of the projectile deceleration. When the underwater projectile speed is very high, the water compressibility effect on the supercavity cannot be ignored. Besides the experiments method, the computational fluid dynamics (CFD) methods and the theoretical methods are used to investigate water compressibility effect. Goncalves et al. [8] used the CFD numerical simulation methods to investigate cavitating flows by taking the vapor phase and liquid phase compressibility into consideration. The pure phase equation of state and the mixture equation of state were presented and the implicit preconditioned compressible RANS solver was used to complete the numerical simulation in [8]. The steady simulation results and unsteady simulation results for global and local cavitation flows analysis are presented by using the compressible wall functions associated with the SST model to model turbulence by Goncalves [9]. Then Goncalves [10] focused on the local compressibility effects on turbulence and described the unsteady behavior of cavity with shedding. Schmidt [11] investigated the cavitating flow through high-speed injection nozzles using the mixture equation of state and Hybrid solver. Saurel et al. [12] introduced a second order Eulerian numerical method to simulate the compressible multifluid flows which allows an arbitrary number of interfaces and very high density ratios. In the past years, many researchers have developed a lot of theoretical methods to investigate the water compressibility effect on the cavity and projectile at high speed underwater motion. Tetsuo et al. [13,14] presented a method to study the supercavitating

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wedge in the subsonic flow based on local linearization and made a comparison with the uniform linearization. Their results showed that the method made a good performance. Chou [15] developed a theory to solve the nonlinear integral differential equation for supercavitating flow based on slender body theory and used it to predict axisymmetric cavities in incompressible flow. Serebryakov [16–18] studied supercavitation in incompressible and compressible flow using asymptotic method. Kuria et al. [19] solved this problem using Chou's theory by applying modified Chebyshev polynomials. Varghese et al. [20] extended Chou's theory and used it to solve cavity shape in subsonic flow. Results of their work showed that the effects of Mach number on cavity shape and surface pressure were roughly opposite to cavitation number effects. Several slender body models comparisons for solving the axisymmetric cavities is present in [20]. Kulkarni et al. [21] and Ohtani et al. [22] have also studied the supercavitation in incompressible flow. Zhang et al. [23] took the first-order approximate solution of cavity shape as the initial solution to start iteration and adopted Riabouchinsky closure based on Varghese's method in subsonic flow. Their results showed good agreement with those obtained by asymptotic method.

The objective of this paper is to investigate the fluid compressibility effect on the cavity shape and the body surface pressure when the underwater vehicle reaches very high speed in the subsonic flow. We propose a novel theoretical model for solving the cavities in axisymmetric flows past a slender conical body in subsonic fluid based on the slender body theory and take account of the fluid compressibility effect in the present paper. A nonlinear integral differential equation is obtained for solving the cavity shape and the body surface pressure. This equation is solved by a numerical discrete scheme and an iterative process. Chou's initial trial solution [15] is extended to the subsonic flow. The critical Mach number is calculated to describe the subsonic flow. The supercavity shapes and hydrodynamic coefficients in incompressible and compressible fluid are obtained by the present theoretical model and compared to the results of other literatures. The compressibility effects on cavity shape, surface pressure distribution and drag coefficient are discussed in the subsonic liquid flow.

2. Mathematical problems

2.1. Governing equation

In order to investigate the cavity behavior when an underwater vehicle reaches very high speed in subsonic flow, a cone-cavity system is introduced here. The cone-cavity system is shown in Fig. 1, and the irrotational and ideal compressible flow is assumed. Let us establish a cylindrical coordinate system (x, r) to describe the cone-cavity system with x along the cone-cavity axis and r perpendicular to x as shown in Fig. 1. The origin of this coordinate system is at the cone nose. The cone and the cavity are axisymmetric, and a conical cavity closure is assumed. The freestream velocity is denoted by u_∞ and the freestream pressure is denoted by p_∞. The cone length is L_b, the cavity length is L_c, and the total length is L. We now normalize all length x, r by L_b, all velocity u, v by u_∞, and velocity potential by $u_\infty L_b$. After normalization, The cone length is 1, the cavity length is l_c, and the total length is l. θ is the half cone angle. The cone-cavity is defined by $r = R(x)$, and $R_{\max}$ is the maximum of $R(x)$. $R(x) = r_b(x) = x\tan\theta$ defines the cone radius and $R(x) = R_c(x)$ defines the cavity. The cavity starts at $x = 1$ and at this point the following boundary conditions are assumed: $r_b(1) = R_c(1)$, $dr_b(1)/dx = dR_c(1)/dx$. The symmetrical Riabushisky scheme for the back of cavity is adopted here.

Assuming φ is the perturbation velocity potential of the flow past the cone, the axial component of the perturbation velocity is $u = \partial\varphi/\partial x$ and the radial component of the perturbation velocity

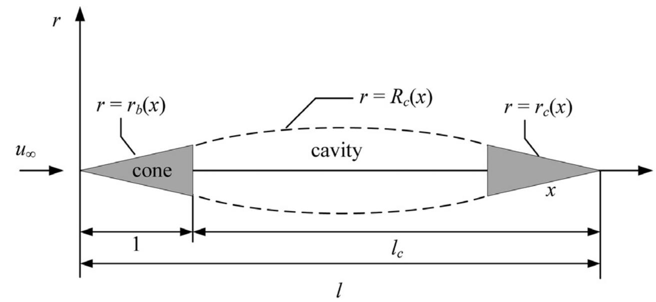

Fig. 1. Slender conical body and supercavity.

is $v = \partial\varphi/\partial r$. Assuming $\delta = R_{\max}/l$, the slender body theory shows that $u = O(\delta^2 \ln\delta)$. The governing equation for the perturbation velocity potential in the subsonic flow is given by:

$$\beta^2 \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = 0 \tag{1}$$

where $\beta = \sqrt{1 - Ma_\infty^2}$ and $Ma_\infty = u_\infty/a_\infty$. Ma_∞ is the Mach number in the freestream and Ma_∞ is smaller than 1 for the subsonic flow. a_∞ is the sound speed in the freestream. The governing equation for the perturbation velocity potential is solved by distributing the source strength $f(x)$ per unit length on the axis x from $x = 0$ to $x = l$. According to Laitone [24], the slender body theory is applied to determine the source strength $f(x)$. We have the source strength $f(x) = \beta S'$, where $S' = dS/dx$ and $S = \pi R^2$. S represents the cross-sectional area of the cone-cavity. The solution of Eq. (1) is given to the first order as:

$$\varphi = -\frac{1}{4}\int_0^l \frac{\frac{dR^2(\xi)}{d\xi}d\xi}{\left[(x-\xi)^2 + (\beta r)^2\right]^{1/2}} \tag{2}$$

The axial component and the radial component of the perturbation velocity can be written as:

$$u = \frac{1}{4}\int_0^l \frac{\frac{dR^2(\xi)}{d\xi}(x-\xi)\,d\xi}{\left[(x-\xi)^2 + (\beta r)^2\right]^{3/2}}$$
$$v = \frac{\beta^2 r}{4}\int_0^l \frac{\frac{dR^2(\xi)}{d\xi}d\xi}{\left[(x-\xi)^2 + (\beta r)^2\right]^{3/2}} \tag{3}$$

2.2. The nonlinear integral differential equation

For water, the Tait's state equation is given by:

$$\frac{p+B}{\rho^\kappa} = \frac{p_\infty + B}{\rho_\infty^\kappa} \tag{4}$$

where $B = 2.98\times10^8$ Pa, $\kappa = 7.15$. p and ρ are the local pressure and fluid density respectively in the flow field. ρ_∞ is the fluid density in the freestream flow. The local sound speed a in the flow past cone satisfies $a^2 = \partial p/\partial\rho = \kappa(p+B)/\rho$. And the sound speed a_∞ in the freestream satisfies $a_\infty{}^2 = \kappa(p_\infty+B)/\rho_\infty$. The compressible Bernoulli's equation for the flow past cone is:

$$\kappa\frac{P+B}{\rho} + \frac{\kappa-1}{2}u_\infty{}^2 q^2 = \kappa\frac{P_\infty + B}{\rho_\infty} + \frac{\kappa-1}{2}u_\infty{}^2 \tag{5}$$

where q^2 is the total speed in the local flow field. The total speed in the local flow field q^2 satisfies the equation that $q^2 = (1+u)^2 + v^2 \approx 1 + 2u + v^2$. After dividing both sides of Eq. (5) by the sound speed in the freestream and some manipulation, we have

$$\left(\frac{P+B}{P_\infty + B}\right)^{\frac{\kappa-1}{\kappa}} + \frac{\kappa-1}{2}Ma_\infty^2\left(2u + v^2\right) = 1 \tag{6}$$