Electric Power Systems Research 97 (2013) 133–143

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A mixed-integer linear programming approach for optimal type, size and allocation of distributed generation in radial distribution systems

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ARTICLE INFO

Article history:
Received 16 August 2012
Received in revised form
14 December 2012
Accepted 17 December 2012
Available online 30 January 2013

Keywords:
Radial distribution system
Distributed generation
Optimal allocation
Mixed-integer linear programming
Short-circuit currents

ABSTRACT

This paper presents a mixed-integer linear programming approach to solving the problem of optimal type, size and allocation of distributed generators (DGs) in radial distribution systems. In the proposed formulation, (a) the steady-state operation of the radial distribution system, considering different load levels, is modeled through linear expressions; (b) different types of DGs are represented by their capability curves; (c) the short-circuit current capacity of the circuits is modeled through linear expressions; and (d) different topologies of the radial distribution system are considered. The objective function minimizes the annualized investment and operation costs. The use of a mixed-integer linear formulation guarantees convergence to optimality using existing optimization software. The results of one test system are presented in order to show the accuracy as well as the efficiency of the proposed solution technique.

1. Introduction

Distributed generation, micro-grid technologies, two-way communication systems, and demand response programs are issues that have been studied in recent years and referred to as *smart grids*. Distributed generation, when it is managed efficiently in technical and economical terms, reduces greenhouse gas emissions, improves efficiency, helps to defer system upgrades, improves reliability, and enhances energy security, among other benefits.

On the other hand, voltage increase at the end of a feeder, demand supply unbalance in a fault condition, power quality decline, increase of power losses and reduction of reliability levels are problems that may occur if the distributed generators (DGs) are not installed properly [1]. In order to solve these problems, an initial approach could be to obtain the optimal location to install the DGs by a complete enumeration of all feasible combinations of sites, types and sizes of the DGs. However, the number of alternatives could be very large as the number of variables of the problem (e.g., number of DGs, types of DGs, number of buses of the system) increases.

Therefore, to solve the problem of optimal allocation of DGs, researchers have employed conventional optimization techniques, such as generalized reduced gradient and Newton–Raphson power flows [2–5]. Analytical approaches using sensitivity factors obtained from quantities, such as the system bus impedance and admittance matrices, and the exact losses formula have also been employed [6–8]. Commercial software, which is composed of linear and non-linear solver tools, is also commonly used [9–12].

Additionally, combinatorial optimization techniques based on artificial intelligence were also used to solve the DGs allocation problem. In [13–16], evolutionary algorithms are used as the solving tool; a tabu search application is presented in [17]; and an ants colony search algorithm was performed in [18]. Although those optimization techniques can bring high quality solutions, they cannot guarantee that those solutions are globally optimal.

The major objectives considered in studies related to the allocation of DGs are to minimize system losses [2–8,14,16,17] and to minimize investment and operation costs [9–13,15,18]. However, further objectives have also been addressed, such as to minimize line loadings [2,16], to improve the voltage profile and the spinning reserve [16], and to maximize the DGs capacity with respect to the technical constraints [12]. The mathematical model of this problem is usually developed through non-linear approaches [2–11,13–15,17,18]. In few studies, for example, in [12,16], the problem is formulated linearly; however, the reactive power balance is not considered, unlike in the present work, where constraints of power balance, both real and reactive power, are considered.

The mathematical models of the DGs are usually viewed through a simple representation where the *coupling element* used to connect the DGs to the network is not detailed; that is, models of Synchronous Generators (SGs), Inverters based on Power Electronics,

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http://dx.doi.org/10.1016/j.epsr.2012.12.009

Table 1
Proposed method compared with other methods.

	Optimization model	Solution technique	Reactive power flows	Coupling elements	Load levels	Scenarios or topologies	Short-circuit currents
Proposed method	MILP	Solver AMPL	•	•	•	•	•
[6]	MINLP	Sensitivity analysis	•	○	•	○	○
[9]	MINLP	Solver GAMS	○	○	○	•	○
[10,11]	MINLP	Solver GAMS	•	○	•	○	○
[12]	MILP	Linear programming algorithm	○	○	○	○	•
[13,14]	MINLP	Evolutionary algorithm	○	○	○	○	○
[19]	MINLP	Sensitivity analysis	•	○	○	•	•
[20]	MILP	Optimization based on weight factors	○	○	○	○	•

•: Yes. ○: No.

Induction Generators (IGs), and Doubly-Fed Induction Generators (DFIGs) are disregarded [2–20]. In most cases, the type of DG (e.g., wind turbine, photovoltaic, biomass-based CHP – Combined Heat and Power – generation systems and small hydroelectric plants) is not specified [6,7,13–15].

Information regarding the DG technology is important since, according to factors such as the *coupling element* used, the operation and impact of the DGs on the system may have different results. For example, it is well known that the capability curve of an IG (commonly used in wind turbines) is different from the capability curve of a SG (generally used in biomass-based CHP generation systems and small hydroelectric plants) [21]. The ability of a DG with an SG to operate both in the reactive absorption area as well as in the reactive supply area may represent different impacts when compared with a DG with an IG, which only absorbs reactive power. DFIGs, which also allow operation in the reactive absorption and supply areas, are also commonly used; however, the capability curve and, consequently, the impact on the network are also different with regard to the IGs and the SGs [21].

Several works take into account the effect of the DGs in the short-circuit levels. In [12], sensitive indexes calculated before the optimization process were used to evaluate the contribution of increasing generation to the short-circuit currents. Another approach uses a multi-objective index that considers the impact of the DGs in the fault currents [19]. In [20], losses, voltage profile and short-circuit levels are used in the algorithm to determine the optimum sizes and locations of the DGs. Commonly, the aforementioned works only considered one topology for the system, that is, they only considered one specific configuration of the circuits that establishes the paths from the substation to the loads, despite the fact that it may change depending on the state of the system.

In Table 1, a review of the main characteristics of the proposed approach against other methods is presented.

In this paper, the problem of optimal type, size and allocation of DGs – which will be henceforth referred to only as the optimal DGs Allocation (DGA) problem – in radial distribution systems is modeled as a Mixed-Integer Linear Programming (MILP) problem. Linearizations were made to adequately represent the steady-state operation of the Electric Distribution System (EDS), considering the behavior of constant power and constant impedance type loads and different load levels. The integer nature of the decision variables represents the allocation, size and type of DGs. Different types of DGs are represented by their capability curves.

The objective of the optimization problem is to minimize the total investment and operation costs subject to operational and physical constraints. The short-circuit current capacity of the circuits is considered and, to account for changes in demand, considerations such as annual variation of the demand (represented through load levels) and different possible topologies for the distribution system are included in the model. In addition, such modeling allows incorporation of probabilistic scenarios for the loads, each with an occurrence probability. The proposed model was tested in a system of 33 buses.

The optimization process is developed from the standpoint of the utility. From this point of view, there are two possibilities:

1. DGs are owned by the utility, in which case the utility is free to allocate them in places it deems advisable according to the solutions of the optimization process.
2. DGs are not owned by the utility, in which case the utility can provide incentives to install DGs in locations that are appropriate for the system, also in accordance with the solutions of an optimization process.

Although DGs are mainly managed by private companies, there are also real applications in which DGs are owned by the utility. As presented in [22], different distributed generation technologies are implemented to fulfill the requirements of a wide range of applications according to the load requirements. In a base load application, DGs owned by the utility are commonly used to provide part of the main required power and support the system by enhancing the voltage profile, reducing the power losses and improving the power quality. According to this definition, our proposal is framed within a base load application, in which DGs are owned by the utility. Therefore, the utility is free to allocate DGs in places it deems advisable for the system.

The proposed method is a useful tool for distribution systems planning, which presents great advantages, in enabling the planner to take appropriate decisions in accordance with the characteristics of the system under study and the economic resources available. It allows the planner to carry out a suitable planning of the distribution system aimed at situations where there are economic opportunities for the installation of DGs for generating power near load centers. The main contributions of this paper are as follows:

1. A novel model for the steady-state operation of a radial EDS through the use of linear expressions.
2. An MILP formulation for the optimal DGA in radial EDSs that presents an efficient computational behavior with conventional MILP solvers. In this formulation, *coupling elements* widely used in distributed generation are considered. In addition, constraints necessary to ensure that DGs allocated in the system will not cause an increase in the short-circuit currents beyond the capacity of the system are also included.

2. The distributed generators allocation problem in radial distribution systems

2.1. Assumptions

In order to represent the steady-state operation of a radial EDS, the following assumptions are made: