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Numerical simulation for Homann flow of a micropolar fluid on a spiraling disk

M. Sajid, M.N. Sadiq *, N. Ali, T. Javed

Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan

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ABSTRACT

In this article the Homann stagnation-point flow of a micropolar fluid over a spiraling disk is considered. A spiraling motion is produced due to uniform rotation and linear radial stretching of the disk. The coupled ordinary differential equations are obtained through the similarity reduction of the governing flow equations of micropolar fluid. A numerical technique known as the shooting method is implemented for obtaining the numerical results. Important features of the flow are investigated for various values of the spiral angle, spiraling parameter and material parameters.

1. Introduction

Due to numerous practical applications in aeronautical science and other branches of engineering such as thermal-power generation, computer storage devices, medical equipments, gas turbine rotors, rotating machinery, air cleaning machines and crystal growth processes, the flow problem over a rotating surface is still being given extraordinary attention by researchers. Therefore, the current study also focuses on the boundary layer flow due to a spiraling disk which combines the effects of linear radial stretching and uniform rotation, but from a new insight unlike to the classical Von Karman flow, that is, in the presence of microelements, which is a hot and emerging topic in the recent literature.

Micropolar fluids belong to a class of the fluids exhibiting the micro-rotational inertia and micro-rotational effects. These fluids possess certain elegance and simplicity in their mathematical modeling which appeals the mathematicians. The micropolar fluids can support couples of the body and couple stress. Certain isotropic fluids, e.g. animal blood, liquid crystals consisting of dumbbell molecules and bar-like elements containing fluids are the examples of micropolar fluids. Other polymeric fluids and the fluids containing the certain additives may be formulated mathematically underlying micropolar fluids.

Stagnation-point flow achieved extensive importance of the researchers working in the field since the stagnation point experiences highest pressure and heat transfer. The pioneering work in this direction was carried out by Heimenz [1]. Stuart [2] extended the Heimenz work by incorporating the external uniform velocity in the viscous fluid flow. Tamada [3] highlighted the effects of obliqueness on the stagnated region in oblique flow on the flat surface. Garg et al. [4] analyzed the numerical simulation for a flow of a second-grade fluid near the stagnation point. Wang [5] analyzed the similarity solution for the flow in the stagnated region of the Navier–Stokes equations.

A famous classical problem for rotary surfaces in fluid mechanics is the swirling flow discussed by Von Karman [6]. The flow over an infinite rotating disk having fluid at rest far away from disk for a viscous fluid, was first investigated by Von Karman. The Navier–Stokes equations were further transformed to a system of nonlinear coupled ordinary differential equations by employing suitable similarity transformations. McLeod [7] discussed the asymptotic solution of the swirling-flow. Zandbergen et al. [8,9] discussed the analytical and numerical solution of Von Karman flow by introducing different families of solution.

The axisymmetric flow in the vicinity a stagnation point towards a flat plate was first discussed by Homann [10]. Wang [11] studied the flow for the surface having radial stretching aligned with or off-centered to stagnation-point of the Homann flow. Hannah [12] discussed the motion of fluid flowing against a rotating disk with axial symmetry. Tifford et al. [13] made an extension of the Hannah's [12] work to estimate the torque on the rotating disk. Wang [14] discussed the off-centered flow in the stagnated region towards a rotating disk. Andersson et al. [15] drew attention towards the flow over the rotating disk of a power-law fluid .

The flow of micropolar fluid towards a stagnation point was obtained by Peddison et al. [16]. Ahmadi [17] discussed the self-similar solution by the variation of the material coefficients. Guram

* Corresponding author.
E-mail address: noveelsheikh@gmail.com (M.N. Sadiq).

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et al. [18] calculated the flow of a micropolar fluid over a rotating disk. Sajid et al. [19] discussed exact solutions of a micropolar fluid for thin film flows. Mehmood et al. [20] discussed the stagnation flow over a lubricated surface for micropolar fluid by introducing a viscoelastic fluid as a lubricant.

Recently Mustafa [21,22] analyzed some flow problems over the stretchable rotating disk. He obtained the solution by employing the numerical spectral method. Weidman [23] presented the stagnation point flow of a Newtonian fluid over a spiraling disk having the combined effects of uniform rotation with the radial stretching. He discussed the flow of a viscous fluid for the Homann stagnation flow and the Argawal stagnation flow on a spiraling disk.

In the present investigation the micropolar fluid is considered over a spiraling disk with the linear stretching and plate rotation. The stagnation-point, uniform rotation and radial plate stretching plays a vital role in the flow. The flow is taken as axisymmetric due to Homann [10].

Following Weidman [23] the present discussion is about the case where the stagnation point, the centers of rotation and of radial stretching are coincident for the flow of a micropolar fluid. This leads to a situation with radial linear stretching and uniform rotation causing the motion at the surface of the plate to form logarithmic spirals. The angle of spiral ϕ is the relative angle to the circles concentric with the axis of rotation. The cases $\phi = 0$ represents the pure rotation and $\phi = 90$ represents the pure radial stretching, respectively.

2. The Homann problem formulation

Consider the Homann [10] stagnation-flow of a micropolar fluid impinging normal to a plate rotating with the angular speed Ω and radially stretching with the strain rate b. The radial and azimuthal velocity components are $u(r,0) = br$ and $v(r,0) = \Omega r$ with anticlockwise rotation ($\Omega \geq 0$). The mathematical model for a micropolar fluid in cylindrical coordinates (r, θ, z) with velocities (u, v, w) and micro-rotations $(N1, N2, N3)$ is given by [18]

$$\frac{1}{r}\frac{\partial}{\partial r}(ur) + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + (\mu + k)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) - k\left(\frac{\partial N_2}{\partial z}\right), \quad (2)$$

$$\rho\left(u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) = (\mu + k)\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right) + k\left(\frac{\partial N_1}{\partial z} - \frac{\partial N_3}{\partial r}\right), \quad (3)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + (\mu + k)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{k}{r}\left(\frac{\partial (rN_2)}{\partial r}\right), \quad (4)$$

$$\rho j\left(u\frac{\partial N_1}{\partial r} - \frac{v}{r}N_2 + w\frac{\partial N_1}{\partial z}\right) = (\alpha + \beta + \gamma)\frac{\partial}{\partial r}\left(\frac{\partial N_1}{\partial r} + \frac{N_1}{r} + \frac{\partial N_3}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial N_3}{\partial r} - \frac{\partial N_1}{\partial z}\right) - k\frac{\partial v}{\partial z} - 2kN_1, \quad (5)$$

$$\rho j\left(u\frac{\partial N_2}{\partial r} + \frac{v}{r}N_1 + w\frac{\partial N_2}{\partial z}\right) = \gamma\left[\frac{\partial}{\partial r}\left(\frac{\partial N_2}{\partial r} + \frac{N_2}{r}\right) + \frac{\partial^2 N_2}{\partial z^2}\right] - k\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right) - 2kN_2, \quad (6)$$

$$\rho j\left(u\frac{\partial N_3}{\partial r} + w\frac{\partial N_3}{\partial z}\right) = (\alpha + \beta + \gamma)\frac{\partial}{\partial z}\left(\frac{\partial N_1}{\partial r} + \frac{N_1}{r} + \frac{\partial N_3}{\partial z}\right) - \gamma\frac{\partial}{\partial r}\left[r\left(\frac{\partial N_1}{\partial z} - \frac{\partial N_3}{\partial r}\right)\right] - \frac{k}{r}\frac{\partial (rv)}{\partial r} - 2kN_3, \quad (7)$$

along with boundary conditions

$$u = br \quad v = \Omega r \quad w = 0 \quad N_1 = -n\frac{du}{dz} \quad N_2 = -n\frac{dv}{dz} \quad N_3 = 0 \quad at \quad z = 0, \quad (8)$$

$$u \to ar \quad v \to 0 \quad w \to -2az \quad N_1 \to 0 \quad N_2 \to 0 \quad N_3 \to 0 \quad at \quad z \to \infty, \quad (9)$$

where ρ, ν and j respectively are density, kinematic viscosity and the gyration parameter of the fluid, p is the pressure, α, β, γ, μ and k are the material constants. Introducing the similarity variables

$$u = arf'(\eta), \quad v = arg(\eta), \quad w = -2\sqrt{a\nu}f(\eta), \quad \eta = \sqrt{\frac{a}{\nu}}z,$$

$$N_1 = a\sqrt{\frac{a}{\nu}}rF(\eta), \quad N_2 = a\sqrt{\frac{a}{\nu}}rG(\eta), \quad N_3 = aH(\eta). \quad (10)$$

where a represents strain rate having units $1/T$ of the Homann stagnation-point flow. So the transformed equations are

$$(1 + k)f'''(\eta) + 2f(\eta)f''(\eta) - f'^2(\eta) + g^2(\eta) - kG'(\eta) + 1 = 0, \quad (11)$$

$$(1 + k)g''(\eta) + 2f(\eta)g'(\eta) - 2f'(\eta)g(\eta) - kF'(\eta) = 0, \quad (12)$$

$$2(1 + k)f''(\eta) + 4f(\eta)f'(\eta) - 2kG(\eta) - P' = 0, \quad (13)$$

$$(1 + k)F''(\eta) - f'(\eta)F(\eta) - 2f(\eta)F'(\eta) + g(\eta)G(\eta) - k\big(g'(\eta) + 2F(\eta)\big) = 0, \quad (14)$$

$$(1 + k)G'''(\eta) + 2f(\eta)G'(\eta) - f'(\eta)G(\eta) - g(\eta)F(\eta) + k\big(f''(\eta) - 2G(\eta)\big) = 0, \quad (15)$$

$$(1 + k)H''(\eta) + 2f(\eta)H'(\eta) + 2k\big(g(\eta) - H(\eta)\big) = 0, \quad (16)$$

associated boundary conditions are

$$f(0) = 0, \quad f'(0) = \lambda, \quad g(0) = S, \quad F(0) = -nf''(0), \quad G(0) = -ng'(0), \quad H(0) = 0, \quad (17)$$

$$f'(\infty) = 1, \quad g(\infty) = 0, \quad F(\infty) = 0, \quad G(\infty) = 0, \quad H(\infty) = 0, \quad (18)$$

where the λ and S appearing in the wall boundary condition are the stretching parameter and the rotation parameter respectively and are defined as

$$\lambda = \frac{b}{a}, \quad S = \frac{\Omega}{a}. \quad (19)$$