European Journal of Mechanics / B Fluids 72 (2018) 364–373

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Hydrodynamic interaction between two rotating spheres in an incompressible couple stress fluid

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ARTICLE INFO

Article history:
Received 8 May 2018
Received in revised form 24 June 2018
Accepted 11 July 2018

Keywords:
Couple stress fluids
Hydrodynamic interaction
Spherical particles
Boundary collocation method

ABSTRACT

This paper focuses on the study of the hydrodynamic interaction between two rotating spheres in an incompressible couple stress fluid. The two spheres are assumed to rotate steadily about the line of their centers with different angular velocities. The general solution for the steady motion of an incompressible couple stress fluid past an axisymmetric particle is obtained analytically in the form of an infinite series. The principle of superposition is utilized to construct the general solution for the steady motion of a couple stress fluid past two rotating spheres using two moving spherical coordinate systems with origins located at the centers of the two spheres. The boundary collocation method is employed to satisfy the imposed boundary conditions on the spherical boundaries. The torque experienced by the fluid on each of the spherical objects is evaluated and represented numerically through tables and graphs. The tabulated results show that the convergence is rapid. In addition, the numerical results show that the increase in the couple stress viscosity parameter increases the values of the normalized torque on each of the two spheres.

1. Introduction

Study of the motion of fluids with microstructure attracts the attention of many researchers to investigate due to its wide area of applications in the fields of chemical, biological and engineering sciences. In addition, the well-known Navier–Stokes theory cannot accurately describe the correct behavior of such types of fluids because it neglects the effects of its microstructure. Motivated by this, Eringen introduced his theory of micropolar fluids to take into consideration the microstructure of such fluids [1]. In the proposed model, the motion of a micropolar fluid is described by two independent vectors; one of them is the classical velocity vector which describes the motion of the macro-volume elements while the other is called microrotation vector which characterizes the motion of the micro-volume elements. These two vectors are satisfying conservation of mass, balance of momentum and balance of angular momentum. In addition to these governing equations, there are two constitutive equations namely stress tensor and couple stress tensor. There are so many research papers discussing the motion of fluids with microstructure using the model of micropolar fluids e.g. [2–6]. This is not the only model proposed to describe the correct behavior of such types of fluid flows. Stokes introduced another theory, namely couple stress fluids, to describe the motion of fluids taking into consideration a size-dependent effect that is not predicted by the classical Navier–Stokes theory [7,8]. In his model, Stokes assumed that the fluid has no microstructure at the kinematic level, so that the classical velocity vector determines the kinematics of motion completely. He also assumed in his theory of couple stress fluids that the surface of a portion of the fluid medium is affected on by a force distribution in addition to a moment distribution. This implies that the constitutive equations needed in the equation of motion consists of two tensors namely stress tensor and couple stress tensor. Therefore, the motion of a couple stress fluid is completely described by a set of two differential equations namely; the equation of continuity and the equation of motion. This equation of motion is similar to the Navier–Stokes equation but with higher order [7,8].

Recently, a growing attention of researchers has been given to the theory of couple stress fluids. The fluid motion of a couple stress fluid representing blood flow in an artery with mild stenosis has been studied by Srivastava [9]. Srinivasacharya and Srikanth [10] investigated the steady motion of a couple stress fluid through a constricted annulus. In [11], Devakar et al. obtained the exact solutions of Couette, Poiseuille and generalized Couette flows of an incompressible couple stress fluid between parallel plates. Ashmawy [12] investigated the unsteady rotational motion of a couple stress fluid around rotating spherical particle under the effect of slip condition. In [13], Srinivasacharya and Rao used the couple stress fluids model to discuss the motion of blood flow through a bifurcated artery. Aparna et al. [14] studied the steady motion

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https://doi.org/10.1016/j.euromechflu.2018.07.005
0997-7546/

of a couple stress fluid past a permeable sphere. The drag force experienced by a steady couple stress fluid flow on a spherical particle moving in it is investigated by Ashmawy [15].

The study of the interaction between spherical particles in an incompressible fluid flow has many practical and industrial applications in natural, biological and industrial processes. Examples of such applications are extraction of proteins and other macromolecules in biological and pharmaceutical procedures, sedimentation, rheology of suspensions, blood cells motion in arteries or veins and water purification processes [16–18]. Motivated by this, many researchers considered various problems of interaction between spherical particles in fluid dynamics. Chen and Keh [17] studied the axisymmetric motion of two spherical particles in the Navier–Stokes theory using slip condition. Faltas et al. [19] investigated the rotational motion of a micropolar fluid past two rotating spheres. Snijkers et al. [20] studied the hydrodynamic interaction between two equal spheres in a viscoelastic fluid. Radiom et al. [21] examined the hydrodynamic interaction between two Brownian spherical particles at small separations and high frequency of thermal oscillations.

This work focuses on the study of the hydrodynamic interaction between two rotating spheres of arbitrary sizes in an incompressible couple stress fluid. The two spheres are assumed to rotate about the line of their centers with arbitrary angular velocities. The problem is solved analytically using two spherical systems of coordinates with origins located at the centers of the two spheres. The boundary collocation technique is then applied to satisfy the boundary conditions on the spherical surfaces. The torque experienced by the fluid flow on each of the spherical particles is obtained and its numerical values are tabulated and graphed.

2. Rotation of axisymmetric particle in a couple stress fluid

Consider the slow steady motion of an incompressible couple stress fluid past an axisymmetric body rotating about its axis of revolution. Taking the assumption of Stokesian flow (low Reynolds number) into consideration and assuming that body forces and body couples are absent, the fluid flow is governed by the following differential equations [8]

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$\eta \nabla \times \nabla \times \nabla \times \nabla \times \vec{q} - \mu \nabla \times \nabla \times \vec{q} + \nabla p = 0, \tag{2.2}$$

where $\vec{q}$ is the velocity vector and p is the fluid pressure at any point. The material constant μ is the classical viscosity coefficient and η denotes the new viscosity coefficient characterizing the effect of couple stress fluid. If this later parameter is taken zero, the equation of motion reduces to that of Stokes equations.

The stress tensor t_{ij} and the couple stress tensor m_{ij} proposed by Stokes [8] are given by

$$t_{ij} = -p\,\delta_{ij} + 2\mu\, d_{ij} - \frac{1}{2} e_{ijk}\, m_{sk,s}, \tag{2.3}$$

$$m_{ij} = m\delta_{ij} + 4(\eta\, \omega_{j,i} + \eta' \omega_{i,j}), \tag{2.4}$$

where m is one third the trace of the couple stress tensor and η' is the second viscosity coefficient characterizing the theory of couple stress fluids.

The couple stress viscosity coefficients, η and η', are satisfying the following inequalities [8]

$\eta \geq 0, \eta \geq \eta'$.

In addition, the deformation rate tensor d_{ij} and the vorticity vector ω_i are defined by

$$d_{ij} = \frac{1}{2}(q_{i,j} + q_{j,i}), \quad \omega_i = \frac{1}{2} e_{ijk}\, q_{k,j}\,. \tag{2.5}$$

The two constant tensors δ_{ij} and e_{ijk} are, respectively, denoting the Kronecker delta and the alternating tensor.

Let us assume that $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi)$ are the unit vectors along the increasing directions of the spherical coordinates (r, θ, φ), respectively. Since the fluid motion is generated by the rotation of an axisymmetric solid of revolution about its axis of symmetry, then the resulting fluid motion is also of axisymmetric nature. Therefore, the velocity and vorticity vectors can be represented, respectively, as

$$\vec{q} = q_\varphi(r,\theta)\vec{e}_\varphi, \tag{2.6}$$

$$\vec{\omega} = \omega_r(r,\theta)\vec{e}_r + \omega_\theta(r,\theta)\vec{e}_\theta. \tag{2.7}$$

The equation of continuity (2.1) is identically satisfied by the velocity vector (2.6) while the momentum equation (2.7) reduces to

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0, \tag{2.8}$$

$$E^2\left(E^2 - \ell^2\right) r \sin\theta\, q_\varphi(r,\theta) = 0, \tag{2.9}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta}. \tag{2.10}$$

In addition,

$$\ell^2 = a^2\mu/\eta, \tag{2.11}$$

where a is denoting the equatorial radius of the axisymmetric body.

The regular solution of the partial differential equation (2.9) appropriate for unbounded fluid region is found to be

$$q_\varphi(r,\theta) = \sum_{n=1}^{\infty}\left(A_n r^{-n-1} + B_n r^{-1/2} K_{n+1/2}(\ell r)\right) P_n^1(\zeta), \tag{2.12}$$

where $\zeta = \cos\theta$.

Moreover, the non-vanishing vorticity components are obtained by using (2.5), (2.7) and (2.12) as

$$\begin{aligned}\omega_r(r,\theta) = \frac{1}{2}\sum_{n=1}^{\infty} n(n+1)\big\{A_n r^{-n-2} \\ + B_n r^{-3/2} K_{n+1/2}(\ell r)\big\} P_n(\zeta),\end{aligned} \tag{2.13}$$

$$\begin{aligned}\omega_\theta(r,\theta) = \frac{1}{2}\sum_{n=1}^{\infty}\big\{nA_n r^{-n-2} + B_n r^{-3/2}\big(nK_{n+1/2}(\ell r) \\ + \ell r K_{n-1/2}(\ell r)\big)\big\} P_n^1(\zeta),\end{aligned} \tag{2.14}$$

where $P_n(.)$, $P_n^1(.)$ and $K_n(.)$ are, respectively, Legendre polynomial of degree n, associated Legendre polynomial of degree n and order 1, and modified Bessel function of the second kind of order n.

The tensor relation (2.4) can be used to give

$$m_{r\theta}(r,\theta) = 4\eta\frac{\partial \omega_\theta}{\partial r} + 4\eta'\left(\frac{1}{r}\frac{\partial \omega_r}{\partial \theta} - \frac{\omega_\theta}{r}\right). \tag{2.15}$$

Inserting the expressions (2.12)–(2.14) into the above expression we get the following explicit form of the tangential couple stress component

$$m_{r\theta}(r,\theta) = -2\sum_{n=1}^{\infty}\Bigg(A_n r^{-n-3}\Big\{n\,\left((n+2)\eta + \eta'\right) P_n^1(\zeta) + \frac{n^2(n+1)^2\eta'}{\sqrt{1-\zeta^2}} G_{n+1}(\zeta)\Big\} + B_n r^{-5/2}\Big\{\Big\{n\,\left((n+2)\eta + \eta'\right) P_n^1(\zeta)$$