European Journal of Mechanics / B Fluids 72 (2018) 432–448

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Irregular wave propagation with a 2DH Boussinesq-type model and an unstructured finite volume scheme

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ARTICLE INFO

Article history:
Received 6 December 2017
Received in revised form 14 June 2018
Accepted 19 July 2018
Available online 26 July 2018

Keywords:
Irregular waves
Wave-breaking
Extended Boussinesq-type equations
Finite volumes
Unstructured meshes

ABSTRACT

The application and validation, with respect to the transformation, breaking and run-up of irregular waves, of an unstructured high-resolution finite volume (FV) numerical solver for the 2D extended Boussinesq-type (BT) equations of Nwogu (1993) is presented. The numerical model is based on the combined FV approximate solution of the BT model and that of the nonlinear shallow water equations (NSWE) when wave breaking emerges. The FV numerical scheme satisfies the desired properties of well-balancing, for flows over complex bathymetries and in presence of wet/dry fronts, and shock-capturing for an intrinsic representation of wave breaking, that is handled as a shock by the NSWE. Several simulations and comparisons with experimental data show that the model is able to simulate wave height variations, mean water level setup, wave run-up, swash zone oscillations and the generation of near-shore currents with satisfactory accuracy.

1. Introduction

Accurate simulations of near-shore hydrodynamics is of fundamental importance to marine and coastal engineering practice. Gravity water waves propagate towards the coastline in groups of low- and high-frequency waves, shoal in shallow waters and eventually break. As such, near-shore hydrodynamics are strongly influenced by the evolution of both low- and high-frequency waves and their interactions. Due to their dispersive nature, these wave groups are transient and evolve in space and time leading to wave focusing that can potentially result to the formation of extreme or rogue waves. Group bound long waves may also be amplified by continuing forcing during shoaling of the short-wave groups in shallower water. In sufficiently shallow water, the short waves within the group break at different depths, leading to further long-wave forcing by the varying breakpoint position. The existence of low-frequency waves on the short wave field is important, as it has been suggested that their presence might lead to the desaturation of the surf zone at short wave frequencies. Furthermore, low-frequency motions contribute significantly to surf and swash zone energy levels which is crucial for a variety of phenomena such as, sedimentation, induced harbor oscillations, and coastal inundation. Thus, modeling satisfactorily these combined physical processes that are present throughout the entire coastal zone, from offshore to the shoreline, is of significant practical importance.

Accurate modeling of low-frequency motion follows from an accurate simulation of breaking-resulted energy dissipation, nonlinear energy transfers and surf–swash zone motion. Important physical effects associated with such nonlinear transformations of waves in near-shore regions can be described by Boussinesq-type equations (BTE). BTE are more appropriate for describing flows in deeper waters where frequency dispersion effects may become more important than nonlinearity by introducing dispersion terms in the modeling thus being more suitable in waters where dispersion begins to have an effect on the free surface. Over the last decades, BTE have been widely used to describe wave transformations in coastal regions. For very recent comprehensive reviews on the theory, numerics and applications of BT models we refer to the review works of [1,2]. The success of the BTE is mainly due to the optimal blend of physical adequacy, in representing all main physical phenomena, and to their relative computational ease, especially when computations in two horizontal dimensions are considered. Extending BTE onshore has been a major challenge for the modeling community since the complex phenomena appearing in the surf- and swash-zone have to be adequately reproduced. As such, appropriate treatment of wave-breaking and wave run-up and run-down in BTE has been proven of crucial importance. Further, the accurate and efficient numerical approximation of BTE is still in the focus of on-going research especially in terms of higher-order discretizations and the adaptive mathematical/numerical description of the flow.

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https://doi.org/10.1016/j.euromechflu.2018.07.009

The first set of extended BTE (so-called standard Boussinesq equations) was derived by Peregrine [3], under the assumption that nonlinearity and frequency dispersion are weak and they are limited to relatively shallower water due to the assumption of weak dispersion. Subsequent attempts to extent the validity and applicability of these standard Boussinesq equations have successfully enhanced their properties and applicability. For example, Madsen and Sorensen [4] and Nwogu [5] have extended the validity of the standard equations by giving a more accurate representation of the phase and group velocities in intermediate waters, closely relating to linear wave theory. Furthermore, significant effort has been made in recent years into advancing the nonlinear and dispersive properties of BT models by including high-order nonlinear and dispersion terms, we refer to [2] and references therein, which in turn are more difficult to (numerically) integrate and thus require substantially more computational effort in their numerical approximation. In extend, substantial research effort has been also devoted to appropriate treatments of the wave-breaking and wave run-up/run-down processes within the BTE framework, we refer for example to [6–16] for such treatments. From the numerical point of view, and due to its inherent conservation properties, the application of the finite volume (FV) approach has become the method of choice in approximating BTE during the past decade, we refer for example in [10,12,13,16–29] among others. However, the application of the FV approach has been applied, in the main, to structured computational grids combined with finite-differences for approximating the dispersion terms in the model equations.

The present work is complementary to [14,30] where, for the first time, a high-order well-balanced unstructured TVD-FV scheme on triangular meshes was presented for modeling weakly nonlinear and weakly dispersive water waves over slowly varying bathymetries, as described by the 2D depth-integrated BTE of Nwogu [5]. The model equations have been consistently derived in [5] from the continuity and Euler equations of motion and are applicable to waves propagating in water of variable depth. They have been derived, using the velocity at an arbitrary distance from the still-water level as the velocity variable instead of the commonly used depth-averaged velocity. In intermediate and deep water, the linear dispersion characteristics of this set of equations are strongly dependent on the choice of the velocity variable. Selecting a velocity close to mid-depth as the velocity variable significantly improves the dispersion properties of the model, making it applicable to a wider range of water depths. The equations do not violate any of the assumptions of Boussinesq theory but simply extends the range of applicability of the equations. Further, these equations admit approximate analytical solitary wave solutions [31] as well as exact solitary solutions e.g. [32–34] which have been used for verification of various numerical schemes in the literature.

The TVD-FV scheme numerically implemented here solves the conservative form of the equations following the median dual node-centered approach, for both the advective and dispersive part of the equations. For the advective fluxes, the scheme utilizes an approximate Riemann solver along with a well-balanced topography source term upwinding. Higher order accuracy in space and time is achieved through a MUSCL-type reconstruction technique and through a strong stability preserving explicit Runge–Kutta time stepping. The numerical model combines the best features of two families of equations: the propagation properties of the BTE and the shock-capturing features of the non-linear shallow water equations (NSWE). At this purpose, it solves the BTE where nonlinear and dispersive effects are both relevant and NSWE where nonlinearity prevails and dispersion is almost negligible. To this end, a new methodology was presented in [14] to handle wave-breaking over complex bathymetries in extended two-dimensional BT models. Certain criteria, along with their proper implementation, were established to characterize breaking waves. Once breaking waves are recognized, a switching is performed locally in the computational domain from the BTE to NSWE by suppressing the dispersive terms in the vicinity of the wave fronts. Thus, the shock-capturing features of the FV scheme enable an intrinsic representation of the breaking waves, which are handled as shocks by the NSWE. An additional methodology was presented on how to perform a stable switching between the BTE and NSWE within the unstructured FV framework. The proposed approach is essential and has been proven efficient, especially in two dimensional conditions, but it has been tested mainly for regular wave propagation. Since the model is intended for practical engineering purposes, it should aim at accurately simulating the global effects of wave-breaking i.e. wave height decay, mean water level setup, current generation, resulting also from irregular wave propagation and inundation. Hence, the model's applicability and validity, with respect to the transformation, breaking and run-up of irregular waves is essential and is the main scope of this presentation. In brief, the main novelties of the present work are

- The implementation of an unstructured well-balanced higher-order FV scheme approximating the BTE of Nwogu for the generation and propagation of irregular waves that intrinsically treats conservatively wet/dry fronts for run-up/run-down computations.
- The application of a robust and efficient numerical wave-breaking treatment, within the FV scheme and for general meshes, that is able to track and resolve the evolution of individual breaking waves and swash zone oscillations as parts of the solution in a 2D flow field.
- The irregular waves generation by the application of an efficient source function technique, adapted to the specific BTE and numerical approach.
- The validation of the numerical approach by testing, extensively, its ability to represent the relevant fundamental phenomena in nearshore regions for irregular wave propagation.

The rest of the presentation is organized as follows. Section 2 presents the model equations solved, namely the extended 2D BT equations of Nwogu written in conservation law form that incorporates the topography effects and friction. The irregular wave generation approach via a source term function is also given in this section. In Section 3 the unstructured finite volume methodology implemented is recalled along with the numerical treatment of breaking waves. In Section 4 we verify the ability of the model to simulate irregular wave evolution as well as surf and swash zone dynamics using characteristic test problems. The numerical results are compared to laboratory measurements involving time-domain data, wave spectra, phase-averaged wave parameters and current fields. Finally, concluding remarks are presented in Section 5.

2. Mathematical model

The model equations solved in the present work, following [14,30], are the extended BTE of [5] which describe weakly nonlinear weakly dispersive water waves in variable bathymetries. The model equations were derived under the assumption that the wave height (A) to constant water depth (h) ratio, $\epsilon := A/h$, which measures the weight of nonlinear effects, and the square water depth to wave length (L) ratio $\mu^2 := h^2/L^2$, which represents the dimension of the dispersive effects, is of the same order with, which lead to a Stokes number $S := \epsilon/\mu^2 = O(1)$. The equations provide accurate linear dispersion and shoaling characteristics for values of kh up to 3 (intermediate water depths), where k is the wave number and kh is essentially a scale of the value of μ, providing a correction of $O(\mu^2)$ to the shallow water theory. By retaining $O(\mu^2)$ terms in the derivation of the models some vertical variations in the horizontal