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Effects of rough boundary and nonzero boundary conditions on the lubrication process with micropolar fluid

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ABSTRACT

The lubrication theory is mostly concerned with the behaviour of a lubricant flowing through a narrow gap. Motivated by the experimental findings from the tribology literature, we take the lubricant to be a micropolar fluid and study its behaviour in a thin domain with rough boundary. Instead of considering (commonly used) simple zero boundary conditions, we impose physically relevant (nonzero) boundary conditions for the microrotation and perform the asymptotic analysis of the corresponding 3D boundary value problem. We formally derive a simplified mathematical model acknowledging the roughness-induced effects and the effects of the nonzero boundary conditions on the macroscopic flow. Using the obtained asymptotic model, we study numerically the influence of the roughness on the performance of a linear slider bearing. The numerical results clearly indicate that the use of the rough surfaces may contribute to enhance the mechanical performance of such device.

1. Introduction

In this paper we investigate the flow of a viscous fluid (acting as a lubricant) between two solid surfaces in relative motion. The lower surface is assumed to be plane, while the upper is described by a given shape function. Such situation appears naturally in numerous industrial and engineering applications, in particular those consisting of moving machine parts (see e.g. [1]). The mathematical models for describing the motion of the lubricant usually result from the simplification of the geometry of the lubricant film, i.e. its thickness. Using the film thickness as a small parameter, a simple asymptotic approximation can be easily derived providing a well-known Reynolds equation for the pressure of the fluid. Formal derivation goes back to the 19th century and the celebrated work of Reynolds [2]. The justification of this approximation, namely the proof that it can be obtained as the limit of the Stokes system (as thickness tends to zero) is provided in [3] for a Newtonian flow between two plain surfaces. More recent results on the lubrication with a Newtonian fluid can be found in [4–6].

The experimental observations from the tribology literature (see e.g. [7–9]) indicate that the fluid's internal structure should not be ignored in the modelling, especially when the gap between the moving surfaces is very small. Among various non-Newtonian models, the model of micropolar fluid (proposed by Eringen [10] in 60's) turns out to be the most appropriate since it acknowledges the effects of the local structure and micro-motions of the fluid elements. Physically, micropolar fluids consist in a large number of small spherical particles uniformly dispersed in a viscous medium. Assuming that the particles are rigid and ignoring their deformations, the related mathematical model expresses the balance of momentum, mass and angular momentum. A new unknown function called microrotation (i.e. the angular velocity field of rotation of particles) is added to the usual velocity and pressure fields. Consequently, Navier–Stokes equations become coupled with a new vector equation coming from the conservation of angular momentum with four microrotation viscosities introduced. Being able to describe numerous real fluids better than the classical (Newtonian) model, micropolar fluid models have been extensively studied in recent years (see e.g. [11–15]).

Engineering practice also stresses the interest of studying the effects of very small domain irregularities on a thin film flow. Indeed, it was observed experimentally and numerically that surface roughness may affect the film thickness and the overall performance in lubricated contact in terms of load carrying capacity, friction and pressure build-up (see for instance [16–18]). Many methods have been developed to model the effect of the roughness by constructing an average Reynolds equation with smooth coefficients, related to the geometry of the asperities.

In the literature, such geometry is described by stochastic or by deterministic tools. Arguing that, due to the inevitable presence of

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random variations at small scales, the exact geometry of a material surface cannot be completely determined by measures, stochastic methods study the impact of statistical properties of the roughness on the averaged behaviour of thin-film lubricants. One may cite the early works of Tzeng and Saibel [19] for one-dimensional transversal roughness, Christensen [20] for one-dimensional longitudinal roughness. In the case of a general rough structure, Patir and Cheng [21] derived a method that rewrites the Reynolds equation in terms of so-called flow factors, obtained by averaging on a control volume. Their method has led to many publications by other authors, and is commonly used in practice, notably because the flow factors can be measured experimentally.

One the other hand, the deterministic approach considers that the roughness geometry of the rough surface is known, and can be described as the graph of a given periodic function. Such geometry corresponds to an engineered surface texture, that presents asperities of a prescribed shape, size and orientation. In practice, those asperities can be manufactured by photoetching [22], laser ablation [23], or using microelectromechanical systems [24]. Many models of lubrication with a deterministic roughness geometry have been studied numerically (see for instance [25,26]). From a mathematical point of view, the deterministic description of the rough surface is related to the introduction of macro and microvariables, and allows for the development of rigorous homogenization methods to derive the average (main order) Reynolds equation by letting the period of the asperities go to zero.

Let us stress that many homogenization methods exist in the literature. For instance, there is currently a strong research effort in the framework of mean field modelling, that includes roughness or non-uniform boundary conditions and relies on the Bruggerman effective medium theory. For more details on this approach, we refer the interested reader to [27] and the references therein.

In the present paper, we will consider a deterministic description of the roughness, and a homogenization method related to the two-scale convergence introduced by Allaire [28]. Let us give more details on the associated homogenization process and on the known results.

In the case of models of thin-film flows of Newtonian fluids, the configuration where the roughness can be described using a given periodic function has been extensively studied. The classical assumption is that the size of the roughness is of the same order as the film thickness, i.e.

$$h_\varepsilon(x) = \varepsilon\, h\left(x, \frac{x}{\varepsilon}\right), \quad 0 < \varepsilon \ll 1.$$

In such setting, the effective model turns out to be the classical Reynolds equation (see e.g. [29,30]) and one needs to compute the correctors in order to detect the roughness-induced effects. The same result is obtained for $h_\varepsilon(x) = \varepsilon\, h\left(x, \frac{x}{\varepsilon^\beta}\right)$ with $\beta < 1$ (see [31]). In view of that, Bresch and co-authors [32] considered in 2010 a new framework, namely

$$h_\varepsilon(x) = \varepsilon\, h\left(x, \frac{x}{\varepsilon^2}\right).$$

As a result, they derived the explicit correction of the Reynolds approximation modifying it at the main order. Moreover, the whole asymptotic expansion (at any order) of the solution has been rigorously derived in [33] providing the optimality with respect to the truncation error. It is important to emphasize that the roughness pattern described by $h_\varepsilon(x) = \varepsilon\, h\left(x, \frac{x}{\varepsilon^\beta}\right)$ with $\beta > 1$ is physically relevant and realistic (see e.g. [34]), and, therefore, has been studied for different situations in recent years.

In the above mentioned papers (see also [35]), the lubricant is assumed to be a classical Newtonian fluid. So far, micropolar fluid film lubrication has been addressed mostly in a simple thin domain with no roughness introduced (see e.g. [36–38]). However, recently, the roughness effects on a thin film flow have been studied as well and new mathematical models have been proposed. Namely, in [39], the authors consider a micropolar flow in a 2D domain, assuming that the roughness is of the same small order as the film thickness, and derive the effective system by using the two-scale convergence method. In [40], a generalized version of the Reynolds equation is derived in the case of a 3D domain with the roughness pattern proposed in [32]. In the previously mentioned references, a zero boundary condition for the microrotation is assumed, implying that the fluid elements cannot rotate on the fluid–solid interface. If s is the horizontal velocity of the boundary, these conditions are written as follows:

$$u = s \quad (u \text{ velocity}), \tag{1}$$

$$w = 0 \quad (w \text{ microrotation}). \tag{2}$$

However, more general boundary conditions for the microrotation were introduced to take into account the rotation of the microelements on the solid boundary. These conditions read

$$w \times n = \frac{\alpha}{2} \operatorname{rot} u \times n\,, \tag{3}$$

where n is a normal unit vector to the boundary. Conditions (3) were effectively proved to be in good accordance with experiments, see [41–44]. The coefficient α describes the interaction between the given fluid and solid; it characterizes microrotation retardation on the solid surfaces.

In [43], a generalized micropolar Reynolds equation is derived by using conditions (1), (3), and the relevance of the new parameter α regarding the performance of lubricated devices for both load and friction, is established by numerical computations. Nevertheless, it was mathematically proved in [45] that it is not possible to consider the boundary condition (3) and simultaneously retain the no-slip condition (1) for the velocity. This would be like considering simultaneously, at the same boundary, a Neumann and a Dirichlet boundary condition. In order to obtain a well-posed variational formulation of the micropolar system, it is straightforward to confirm (see e.g. [45]) that a velocity condition compatible with (3) needs to be introduced. This condition allows a slippage in the tangential direction and retains a non-penetration condition in the normal direction n (δ_0 is a real parameter)

$$(u - s) \times n = \delta_0 \operatorname{rot} w \times n, \quad u \cdot n = 0. \tag{4}$$

It is worth stressing that in most lubrication studies, it is assumed that the speed of the lubricant at the surface equals that of the solid surface. However, it has been found that wall slip occurs, not only in non-Newtonian flows [46–51], but also in hydrodynamic lubrication or elasto-hydrodynamic lubrication [52–55]. It seems that such phenomenon is linked to physical or chemical interactions of the solid surfaces with the lubricant. Several boundary conditions have been considered in those works to model the observed slippage. Most of them include limited yield stress or retain slippage value proportional to the shear stress. In that context, condition (4) appears as a new interpretation of the slippage observed in lubrication with micropolar fluids, expressed in terms of the microrotation field w.

In [45], by using the nonstandard boundary conditions (3)–(4) described above, in a 2-dimensional thin domain without roughness (see also [56] for the 3D flow), Bayada et al. derive rigorously a generalized version of the Reynolds equation taking such boundary conditions into account. They perform their study in the critical case where one the non-Newtonian characteristic parameters of the micropolar fluid has specific (small) order of magnitude. The authors provide numerical results, focusing on the influence of the slippage on the performance of a linear slider bearing. In comparison with the model in [43] that uses the no-slip condition (1) for