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# Double-deck structure in the problem of a compressible flow along a plate with small localized irregularities on the surface ☆

R.K. Gaydukov

*National Research University Higher School of Economics, Russian Federation*

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ABSTRACT

We consider the stationary problem of flow of a viscous compressible subsonic fluid along a flat plate with small localized (hump-type) irregularities on the surface for large Reynolds numbers. We obtain a formal asymptotic solution with double-deck structure of the boundary layer. We present the results of numerical simulation of the flow in the thin boundary layer (i.e., in the near-boundary region).



## 1. Introduction

The double-deck structure of the boundary layer is encountered in various problems of flow of an incompressible viscous fluid along various surfaces with small irregularities for large Reynolds numbers. For example, in problems of flow past a semi-infinite plate with periodic [1,2] or localized [3] irregularities, in pipes and channels with periodic irregularities on the walls [4], and in problems of flow of submerged jets along a plate with localized irregularities [5].

The double-deck structure was also found in the problem of flow of a compressible fluid along a plate with periodic irregularities [6]. The goal in this paper is to study a more "classical" problem, i.e., the problem of flow along a localized hump-type irregularity on a semi-infinite plate in the case of compressible flows. It will be shown that the solution of this problem also has a double-deck structure of the boundary layer.

Generally speaking, the double-deck structure has been discovered comparatively recently. The classical solution of the flow problems (but on other scales) is the well-known triple-deck structure, which has been widely studied by F.T. Smith, K. Stewartson, O.S. Ryzhov, A.I. Ruban, V.Ya Neiland, and others (see, e.g., [7–16]). The double-deck structure was also considered in the literature for various problems (for example, see [1–6,17,18]), but not as widely as the triple-deck structure.

The main difference between the double-deck and triple-deck structures consists in the following. In the double-deck structure, the flow perturbation due to irregularities on the surface occurs in the classical Prandtl boundary layer region and does not influence the external flow. Hence, the problem of interaction between the boundary-layer flow and the external flow does not appear in case of the double-deck structure in contrast to the triple-deck structure.

However, in contrast to the triple-deck structure (see, e.g., [10]), the double-deck structure has not been studied in detail for compressible fluids. The main goal in this paper is to generalize the double-deck structure to the case of compressible fluids.

We consider a steady-state viscous compressible fluid flow along a flat semi-infinite plate with small hump-type irregularity localized at $x_0$ on the surface for a large Reynolds number $\mathbf{Re}$, see Fig. 1.

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*E-mail address:* rgaydukov@hse.ru.

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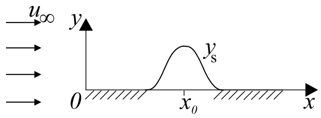


**Fig. 1.** Hump on the plate.

This problem is described by the stationary Navier–Stokes system for compressible fluid (see [19]):

$$\begin{cases} \rho\langle \hat{\mathrm{U}}, \nabla\rangle \hat{\mathrm{U}} = -\nabla \hat{p} + \eta \Delta \hat{\mathrm{U}} + \eta \nabla \langle \nabla, \hat{\mathrm{U}}\rangle /3, \\ \langle \nabla, \hat{\rho}\hat{\mathrm{U}}\rangle = 0, \end{cases} \tag{1}$$

where $\hat{\mathrm{U}} = (\hat{u}, \hat{v})$ is the velocity vector, $\hat{p}$ is the pressure, $\eta$ is the dynamic viscosity, $\hat{\rho}$ is the density, and $(\hat{x}, \hat{y})$ are the coordinates. We suppose that the upstream flow is plane-parallel with velocity $\hat{\mathbf{U}}_\infty = (\hat{u}_\infty, 0)$ and density $\hat{\rho}_\infty$.

We pass to the dimensionless form of nonstationary Navier–Stokes equation (1). Setting dimensionless variables (without hat) as $(x, y) = (\hat{x}/L, \hat{y}/L)$, $p = \hat{p}/P$, $\mathrm{U} = (u, v) = (\hat{u}/u_0, \hat{v}/u_0)$, $\rho = \hat{\rho}/\rho_0$, we obtain

$$\begin{cases} \rho\langle \mathrm{U}, \nabla\rangle \mathrm{U} = -\nabla p \dfrac{P}{\rho_0 u_0^2} + \dfrac{1}{\mathbf{Re}}\big(\Delta \mathrm{U} + \nabla\langle \nabla, \hat{\mathrm{U}}\rangle/3\big), \\ \langle \nabla, \rho \mathrm{U}\rangle = 0, \end{cases} \tag{2}$$

where $L$ is the characteristic length, $u_0$ is the characteristic speed, $\mathbf{Re} = \frac{\rho_0 u_0 L}{\eta}$ is the Reynolds number. Obviously, setting $P = \rho_0 u_0^2$ and $\varepsilon = \mathbf{Re}^{-1/2}$, we obtain

$$\begin{cases} \rho\langle \mathbf{U}, \nabla\rangle \mathbf{U} = -\nabla p + \varepsilon^2(\Delta \mathbf{U} + \nabla\langle \nabla, \mathrm{U}\rangle/3) \\ \langle \nabla, \rho\mathbf{U}\rangle = 0. \end{cases} \tag{3}$$

In the dimensionless form, the upstream flow is $U_\infty = (u_\infty, 0)$, $\rho_\infty$, where $u_\infty = \hat{u}_\infty/u_0$ and $\rho_\infty = \hat{\rho}_\infty/\rho_0$.

We assume that the plate surface is described by the relation

$$y_s = \varepsilon^{4/3}\mu(\xi), \tag{4}$$

where $\xi = (x - x_0)/\varepsilon$, $\varepsilon = \mathbf{Re}^{-1/2}$ is a small parameter, and $\lim_{\xi\to\pm\infty}\mu(\xi) = 0$.

As will be shown below, the asymptotic solution of problem (3), (6) has a double-deck structure, which consists of a thin boundary layer and the classical boundary layer, see Fig. 2.

For simplicity, we assume that

$$p = K\rho, \qquad K = \mathrm{const} > 0, \tag{5}$$

where $\sqrt{K}$ is the speed of sound. We note that only the subsonic flow ($u_\infty < \sqrt{K}$) is considered in this paper.

We also note that all possible flows are localized near the irregularity and do not influence the flow at a far distance from it.

The boundary conditions are

$$\mathbf{U}\Big|_{\substack{y = y_s \\ x > 0}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \frac{\partial u}{\partial y}\Big|_{\substack{y = 0 \\ x < 0}} = 0, \ v\Big|_{\substack{y = 0 \\ x < 0}} = 0,$$
$$\mathbf{U}\Big|_{y\to\pm\infty} \to \begin{pmatrix} u_\infty \\ 0 \end{pmatrix}, \ \mathbf{U}\Big|_{x\to-\infty} \to \begin{pmatrix} u_\infty \\ 0 \end{pmatrix}. \tag{6}$$

We note that problem (3), (6) was considered earlier in the incompressible case in [1,2].

We also note that this problem was investigated in the compressible case but for periodic irregularities on the surface in [6]. The main difference is that, in the periodic case in contrast to the localized case, the solution separates into two components, i.e., the oscillating and averaged parts.

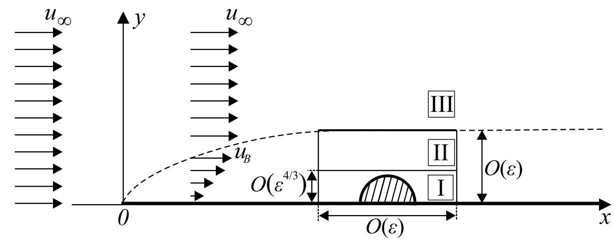


**Fig. 2.** Double-deck structure of the boundary layer.

## 2. Formal asymptotic solution

As was already mentioned, the solution of problem (3)–(6) under study has the double-deck structure (see Fig. 2). We introduce the scale

$$\theta = y/\varepsilon^{4/3}, \quad \tau = y/\varepsilon, \quad \xi = (x - x_0)/\varepsilon.$$

The superscript on the functions below stands for the number of the deck: I is the thin boundary layer (variables $(\xi, \theta)$), II is the classical boundary layer (variables $(\xi, \tau)$), and III is the external region.

We assume that the presence of a hump does not affect the flow far from it (i.e. all functions depending on $\xi$ tend to zero as $\xi \to \pm\infty$). Also note that all functions below depending on $\tau$ $(\theta)$ decrease by the law $|\tau^{-N}|$ $(|\theta^{-N}|)$ as $\tau \to \infty$ $(\theta \to \infty)$, where $N \in \mathbb{N}$ is sufficiently large, see [3] for details.

The formal asymptotic solution of problem (3), (6) has the form

$$\begin{aligned} u(x, y) &= u_\infty + u_0^{\mathrm{II}}(x, \tau) + \varepsilon^{1/3}u_1^{\mathrm{I}}(x, \xi, \theta) \\ &\quad + \varepsilon^{2/3}u_2^{\mathrm{II}}(x, \xi, \tau) + \mathcal{O}(\varepsilon), \\ v(x, y) &= \varepsilon^{2/3}\big(v_2^{\mathrm{I}}(x, \xi, \theta) + v_2^{\mathrm{II}}(x, \xi, \tau)\big) + \mathcal{O}(\varepsilon), \\ \rho(x, y) &= \rho_\infty + \varepsilon^{2/3}\rho_2^{\mathrm{II}}(x, \xi, \tau) + \mathcal{O}(\varepsilon), \end{aligned} \tag{7}$$

where $\theta = y/\varepsilon^{4/3}$, $\tau = y/\varepsilon$, $\xi = (x - x_0)/\varepsilon$, $u_\infty$ is the velocity of the plane-parallel upstream flow, $\rho_\infty$ is its density, and $u_0^{\mathrm{II}} = u_0^* - u_\infty$.

The function $u_0^*$ has the form

$$u_0^* = u_\infty f'\big(\tau/\sqrt{x}\big),$$

where $f(\gamma)$ is the solution of the Blasius-type problem

$$\begin{cases} \rho_\infty f f'' + 2f''' = 0, \\ f(0) = f'(0) = 0, \quad f'(\infty) = 1. \end{cases} \tag{8}$$

This system of equations is an analogue of the Blasius equation for incompressible flow in classical Prandtl boundary layer, and the difference is in the presence of the coefficient $\rho_\infty$, see [20,21].

The functions $u_1^{\mathrm{I}}$ and $v_2^{\mathrm{I}}$ are

$$u_1^{\mathrm{I}} = u_1^* - \theta \frac{\partial u_0^*}{\partial \tau}\Big|_{\tau=0}, \quad v_2^{\mathrm{I}} = v_2^* - v_2^{\mathrm{II}}\big|_{\tau=0},$$

where the functions $u_1^*$ and $v_2^*$ satisfy the problem

$$\begin{cases} \rho_\infty\Big(u_1^*\dfrac{\partial u_1^*}{\partial \xi} + v_2^*\dfrac{\partial u_1^*}{\partial \theta}\Big) + K\dfrac{\partial \rho_2^{\mathrm{II}}}{\partial \xi}\Big|_{\tau=0} - \dfrac{\partial^2 u_1^*}{\partial \theta^2} = 0, \\ \dfrac{\partial u_1^*}{\partial \xi} + \dfrac{\partial v_2^*}{\partial \theta} = 0. \end{cases} \tag{9}$$

$$u_1^*\big|_{\theta=\mu} = \mu\frac{\partial u_0^*}{\partial \tau}\Big|_{\tau=0}, \quad v_2^*\big|_{\theta=\mu} = 0. \tag{10}$$

$$\frac{\partial u_1^*}{\partial \theta}\Big|_{\theta\to\infty} \to \frac{\partial u_0^*}{\partial \tau}\Big|_{\tau=0}, \quad \frac{\partial u_1^*}{\partial \xi}\Big|_{\theta\to\infty} \to 0, \tag{11}$$