Spatial instability of double-layer viscoelastic liquid sheets in a viscous gas medium

Hao-lin Li, Li-zi Qin, Han-yu Ye, Li-jun Yang *

School of Astronautics, Beihang University, Beijing 100191, China

A R T I C L E   I N F O

A B S T R A C T

The spatial instability of double-layer viscoelastic liquid sheets in a viscous gas medium is investigated. A linear stability analysis is used to simplify the governing equations and boundary conditions combined with the Oldroyd-B model for describing the viscoelastic liquid. The growth rate and the cut-off wavenumber have been studied by analyzing the dispersion curve which is based on the spectral collocation method. Moreover, we use the decomposition of growth rate to interpret some special cases. We conclude that the thickness ratio, the liquid density ratio, the liquid viscosity ratio, and the gas-to-liquid viscosity ratio stabilize the liquid sheets, while the gas-to-liquid density ratio, the Reynolds number, and the Weber number destabilize the liquid sheets. The dispersion curves are nearly-identical when the surface tension ratio is smaller than one, while the surface tension ratio is a stabilizing factor when the ratio is larger than one, but the second maximum and the counterintuitive destabilization exist in the large wavenumber region. As specific parameters concerning the viscoelasticity, the elasticity number transforms from the destabilizing factor into the stabilizing factor with the increased Reynolds number in the region nearby the cut-off wavenumber, because the relationship between the change rates of viscous dissipation terms and elasticity terms in the decomposition of the growth rate is varying. Besides, it is discovered that the trend of the change of the growth rate under the varying stress relaxation time ratio is non-monotonic and double-layer liquid sheets are most stable when the stress relaxation time ratio is near 3 (not 1), because the difference between the change rate of the viscous dissipation and the deformation retardation dissipation determines the most stable region (not the elasticity difference), which can also be interpreted with the variation of the energy terms.

© 2018 Elsevier Masson SAS. All rights reserved.

1. Introduction

It is generally known that the thin sheet is one of the main forms in which a liquid issues from an orifice. Due to the wide range of the practical applications of the atomization of sheets, such as spray combustion, agricultural sprays, film coating chemicals, and pharmaceutical processing. It is pivotal to study the mechanisms of instability and breakup of the liquid sheet. Hence the liquid sheet stability has been researched as a classical problem for a long time by many researchers.
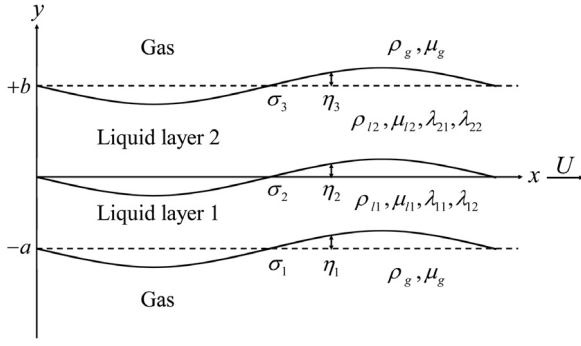
The stability of an inviscid liquid sheet moving in an inviscid stationary gas medium was first investigated by Squire [1]. He studied the temporal linear stability of an inviscid liquid sheet in an inviscid gas medium and concluded that the sheet is unstable if the Weber number is larger than one. He also found the dimensionless wavenumber for maximum instability, which has been found to be an important parameter. The further study by Hagerty & Shea [2]

showed that only two modes of waves can exist, sinuous waves (antisymmetric) and varicose waves (symmetric). Especially, the growth rate of the former is greater than that of the latter. Also, the experiments were performed for the comparison with the theoretical predictions. With the progress in the linear stability analysis of the inviscid liquid sheet, more researchers began to study the more perfect model of the sheet by adding more physics to the analysis. For example, the linear stability of the viscous sheet has been investigated by Dombrowski and Johns [3], Lin [4], Li and Tankin [5], and Li [6]. They concluded that the instability of the liquid sheet is dominated by a velocity difference between liquid and gas. Their study also showed that the viscosity of the liquid sheet plays a delicate dual role on sheet instability in association with the Weber numbers.

It should be noted that all the models established above are confined to the classic single layer liquid sheet. The double-layer liquid sheets may be involved in some applications, such as multilayer curtain coating (Dyson [7]) and doublet impinging-jet injectors (Sutton and Biblarz [8] and Zhao et al. [9]). To explain the instabilities of double-layer viscous liquid sheets, Ye et al. [10] conducted a

Fig. 1. Schematic diagram of the two-dimensional double-layer viscoelastic liquid sheets moving in a stationary gas medium.

linear stability analysis and considered two situations, an inviscid gas situation and a viscous gas situation. The analytical dispersion was derived in the inviscid gas situation, which indicated that the cutoff wavenumber of the dispersion curve is larger than that of a single-layer sheet. While in the viscous gas situation, the dispersion was more difficult to acquire because of the existence of the gas boundary layer. Hence Ye et al. used a spectral collocation method to solve the problem. It is significant that the growth rate at large wavenumber is fairly suppressed in comparison with the inviscid gas solution.

Although there is a detailed analysis of the instability regarding the viscous double-layer liquid sheets, similar research on the double-layer viscoelastic liquid sheets is lacking. The aim of this study is to find out how differently the instability occurs for the double-layer viscoelastic liquid sheets. The Oldroyd-B model is used to describe the viscoelastic liquid. The breakup model of the double-layer viscoelastic liquid sheets is explained in Section 2. The dispersion curves and the relevant analysis of the instability are presented in Section 3. The findings are summarized in Section 4.

2. Model and method

Fig. 1 presents the model of a two-dimensional double-layer viscoelastic liquid sheets moving in a stationary gas medium.

The displacements of perturbations and the surface tensions of the three interfaces are η_j and σ_j, where $j = 1, 2, 3$ represents the bottom, the middle, and the top interface. Note that the surface tension of the middle interface is approximately $\sigma_2 = |\sigma_3 - \sigma_1|$ (Hertz and Hermanrud [11]).

In the liquid layers, the thicknesses of the first and second liquid layers are a and b, respectively. The densities, viscosities, perturbation velocities in the x and y directions, and perturbation pressures are denoted by ρ_{lj}, μ_{0j}, u_j, v_j, and p_j, where $j = 1, 2$ represents the liquid layer 1 and the liquid layer 2. Besides, the components of the extra stress tensors are τ_{xxj}, τ_{xyj}, and τ_{yyj}, where $j = 1, 2$ represents the liquid layer 1 and the liquid layer 2.

In the gas area, the basic flow in the liquid is uniform and the velocity of the both layers is U. The density and viscosity of the surrounding gas medium are ρ_g and μ_g, respectively. In addition, the perturbation velocities in the x and y directions and perturbation pressures are u_{gj}, v_{gj}, and p_{gj}, where $j = 1, 2$ represents the lower gas and the upper gas.

Due to the viscosity of the gas medium and the no-slip condition at the liquid–gas interfaces, the modified Stokes layer model in the work of Tammisola et al. [12] is adopted because the model is in good agreement with experiments. For the modified Stokes layer model, the air velocity is expressed by the following equation:

$$U_g = U - U \text{erf} (\lambda / 2) \tag{1}$$

where erf() is the error function. λ has the different forms in two different regions as follows:

For the upper gas region,

$$\lambda = (y - b) / \sqrt{\nu_g [T]} \tag{2}$$

For the lower gas region,

$$\lambda = (-a - y) / \sqrt{\nu_g [T]} \tag{3}$$

where ν_g is the kinematic viscosity of the gas,

$$\nu_g = \mu_g / \rho_g \tag{4}$$

and $[T] = \frac{x}{U}$ is a function of x meaning the growth of the gas boundary layer in the streamwise direction. Especially, $x = 0$ corresponds to the position where the liquid sheet is ejected into the surrounding gas.

We choose the Boger fluids as the specific research object because this kind of viscoelastic fluid has the nearly constant viscosity, which is beneficial for the researchers to perform the relevant experiment to study the elastic effects separated from viscous effects. Under small shear rates, the rheological equation of the Boger fluid can be characterized by the Oldroyd-B model. Thus the viscoelasticity is described with the Oldroyd-B model as follows [13]:

$$\tau + \lambda_1 \frac{D\tau}{Dt} + \frac{1}{2}\mu_0 (tr\tau) \dot{\gamma} - \frac{1}{2}\mu_1 \{\tau \cdot \dot{\gamma} + \dot{\gamma} \cdot \tau\} - \frac{1}{2}\nu_1 (\tau : \dot{\gamma}) \delta$$
$$= -\mu_0 \left[\dot{\gamma} + \lambda_2 \frac{D\dot{\gamma}}{Dt} - \mu_2 \{\dot{\gamma} \cdot \dot{\gamma}\} - \frac{1}{2}\nu_2 (\dot{\gamma} : \dot{\gamma}) \delta \right], \tag{5}$$

and

$$\begin{cases} \dot{\gamma} = \nabla v + (\nabla v)^T, \\ \omega = \nabla v - (\nabla v)^T, \\ \dfrac{D\tau}{Dx} = \dfrac{\partial \tau}{\partial t} + (v \cdot \nabla) \tau + \dfrac{1}{2} \{\omega \cdot \tau - \tau \cdot \omega\}, \\ \dfrac{D\dot{\gamma}}{Dx} = \dfrac{\partial \dot{\gamma}}{\partial t} + (v \cdot \nabla) \dot{\gamma} + \dfrac{1}{2} \{\omega \cdot \dot{\gamma} - \tau \cdot \dot{\gamma}\}, \end{cases} \tag{6}$$

where $\dot{\gamma}$ is the strain tensor, ω is the vorticity tensor, μ_0 is the zero shear viscosity, λ_1 is the stress relaxation time, λ_2 is the deformation retardation time, D/Dt is defined as the co-rotational derivative, μ_1, μ_2, ν_1, and ν_2 are the constants which are not fixed (usually with $\nu_1 = \nu_2 = 0$, and often with $\mu_1 = \lambda_1$, $\mu_2 = \lambda_2$). Neglecting the nonlinear terms and gravitation effects, the following linearized equation can be established:

$$\tau + \lambda_1 \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \tau = -\mu_0 \left[\dot{\gamma} + \lambda_2 \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \dot{\gamma} \right]. \tag{7}$$

In our work, λ_{j1} and λ_{j2} denote the stress relaxation time and deformation retardation time of the two liquid layers, where $j = 1, 2$ represents the liquid layer 1 and the liquid layer 2.

Because of the small perturbation in the initial stage of the destabilization process, the linear stability analysis is used to simplify the equations. The linearized governing equations can be established:

Liquid layer 1,

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \text{ for } -a < y < 0, \tag{8}$$

$$\rho_{l1} \left(\frac{\partial u_1}{\partial t} + U \frac{\partial u_1}{\partial x} \right) = -\frac{\partial p_1}{\partial x} + \frac{\partial \tau_{xx1}}{\partial x} + \frac{\partial \tau_{xy1}}{\partial y}$$
$$\text{for } -a < y < 0, \tag{9a}$$