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Slow viscous flow past cylindrical particles with thin liquid layer: Cell model

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ABSTRACT

In order to investigate the hydrodynamic interaction between an interface and a cylindrical particle, it is essential to compute the drag and other physical properties exerted on the cylinder in the vicinity of the interface. The present study examines the analytical solution of slow viscous flow past a swarm of cylindrical particles, where each particle consists of a solid core covered by a liquid shell coated with thin monomolecular layer of surfactant using cell model technique. We have assumed that each particle is enclosed by a hypothetical cell. The boundary conditions of Happel, Kuwabara, Kvashnin and Mehta–Morse models are considered on hypothetical cell. The effects of Surfactants are accommodated by considering the Scriven boundary conditions at the shell involving Surface Shear viscosity and Surface Dilatational viscosity. The variation of drag force and hydrodynamic permeability with various parameters is graphically presented. The result reveals in all cases the strong influence of surface viscosity on the motion of cylindrical particle, following cell model technique. It is seen that the presence of surfactant layer increases drag force on the contrary decreases hydrodynamic permeability. In the limiting cases, the analytical solutions describing the drag force reduces to known results of Happel, Kuwabara, Kvashin and Mehta-Morse.

1. Introduction

A number of models have been developed over the past five decades to describe flow through monomolecular surfactant layer. The celebrated paper of Boussinesq [1], first introduced the idea of surface shear viscosity and surface dilatational viscosity that play an important role in the investigation of fluid–fluid interfacial behavior. The work by Sternling [2], together with Scriven [3] paper, wherein Boussinesq [1] theory was generalized to material interfaces of arbitrary curvature, was a remarkable turning point toward today's modern understanding of interfacial rheology [cf. [4]]. These early studies were the basis for developing various models and measuring techniques in rheology as well as in interfacial rheology. The requirements of modern technology have stimulated the interest in the study of multiphase flow involving the interaction of interfacial phenomena. Theoretical and experimental studies are found in the literature dealing with the analysis of surfactant. This topic has generated a lot of attention amongst mathematician, physicists and engineers due to its manifold application to subjects like lubrication, chemical engineering, colloidal science and biomedical applications. Hence the literature pertaining to surfactant layer is vast and keeps growing continuously; to cite a few cases, we may mention the papers of Goodrich [5–7], Davis [8–11], Shail [12], Lee [13], Yang [14], Sadhal [15], Chakrabarti [16], Schneider [17], O'Neil [18] who have worked on the motion of a body straddling the interface between two immiscible fluids. Other notable contributions which helped in the growth of the subject of surfactant were by Stone [19], Blawzdziewicz [20], Li [21], Zhang [22], Datta [23], Johnson [24], Harper [25], Alves [26], Takemura [27], Tiwari [28], Xu [29], Mason [30], Datta [31] and many more.

All surface active agents are to some extent soluble in the bulk liquid, which in turn signifies that a complete dynamical model needs to describe the motion of the bulk liquid, bulk concentration of surfactant, the surface concentration of surfactant and the adsorption/desorption processes that exchange surfactant molecules between the bulk liquid and the free surface. However, for practical importance, the bulk solubility of surfactants is actually very small. Then we can consider "insoluble surfactants" and only considers the dynamics of the surfactant that is adsorbed at the free surface. Dynamic properties of interfaces are of increasing interest in science as they give an insight into the process and properties of interfaces leading to diverse applications in biosciences and chemical industry. Two types of surfactants exist, soluble and insoluble. From a mathematical point of view the main difference

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between the two types is that in the case of soluble surfactants an extra transport equation for the concentration of surfactants has to be solved in the bulk phases. When the surfactant is insoluble in the bulk phases, so that the bulk transport of surfactant can be neglected, the mathematical description of the problem is completed by the addition of a time - dependent convective–diffusion equation governing the distribution of surfactant on the surface of the drop/bubble. Here an insoluble surfactant is considered where the surface tension is assume to be constant, hence neglected the Marangoni effect. In this case the flow is calculated in the frame of Newtonian volume with surface viscosity (Shear and Dilatational), not taking into account the equations for the surfactants mass balance. The computations are carried for low Reynolds and Capillary number and result are presented for different values of surface shear and dilatational viscosities. Danov [32–34] have worked a lot on the effect of surface viscosities present due to surfactant in different flow problems.

This paper is concerned with the study of incompressible Newtonian fluid past swarm of cylindrical particles coated with thin monomolecular layer of surfactant. In many practical applications, particles are not isolated hence it is necessary to determine whether the presence of neighboring particles or boundaries significantly affect the dynamics of the particles. But in mathematical analysis it is cumbersome to study the motion of swarm of particles through a liquid. In a way to avoid such difficulty cell model is used widely. In cell model technique, the particles are uniformly distributed throughout the fluid phase and every particle is enclosed in the cell formed by the disperse medium. The basic principle of the cell model technique is to replace a system of randomly oriented particles by a periodic array of spheres or cylinders embedded in identical spherical or cylindrical liquid cells. Appropriate boundary conditions on the cell take into account, the effect of surrounding particles on the particle in the center of the cell. Hence, cell model can be used to reduce the solution of the boundary-value problem for the flow around a system of particles to the problem for a single particle.

Happel [35,36] and Kuwabara [37] proposed cell models in which both particle and outer envelope are spheres/cylinders. The advantage of such formulation is that it leads to an axially symmetric flow that has a analytical solution. Happel's model assumed that shear stress vanishes on the outer cell boundary, while according to the Kuwabara's model vorticity vanishes on the outer cell boundary. The hydrodynamic interpretations of both the models are much different. The Happel's model does not require an exchange of mechanical energy between the cell and the environment, while the Kuwabara's model requires an exchange of mechanical energy with the environment. Two other cell models have been suggested by Kvashnin [38] and Cunningham [39] (and later by Mehta [40]) using different boundary conditions on the outer boundary of the cell. Kvashnin [38] proposed the condition that the tangential component of velocity reaches a minimum at the cell surface with respect to radial distance, signifying the symmetry on the cell. Cunningham [39]/Mehta [40] assumed the tangential velocity as a component of the average fluid velocity, signifying the homogeneity of the flow on the cell boundary. The importance of Cunningham/Mehta–Morse boundary condition is that we average the flow variables on a small scale over a cell volume to obtain the large scale behavior.

Analytical solutions of particle-in-cell models are always useful to many practical problems and many authors have worked in this direction. Stechkina [41] evaluated the drag force experienced by porous cylinders in a viscous fluid at low Reynolds number. Pop [42] reported an analytical study of the steady incompressible flow past a circular cylinder embedded in a constant porosity medium based on the Brinkman model. Stokes flow past a swarm of porous circular cylinders with Happel and Kuwabara conditions was discussed by Deo [43]. The flow around nanospheres and nanocylinders was investigated by Matthews [44] and they employed a boundary condition that attempts to account for boundary slip due to the tangential shear at the boundary using a slip length parameter. Deo [45] studied the problem of Stokes flow through a swarm of porous circular cylinder-in-cell enclosing an impermeable core with Kuwabara's boundary condition. Recently, Vasin [46] have calculated the hydrodynamic permeability of a set of impenetrable cylindrical particles covered with a porous layer. The cases of transverse, longitudinal, and random orientations of cylinders relative to the liquid flow were considered using all the four well known boundary condition (Happel, Kuwabara, Kvashnin and Mehta–Morse) on the cell surface and Yadav [47] have calculated that of a biporous membrane.

Abundant literature is present on particle-in-cell model having particle covered with a porous layer. In a similar manner, many authors have investigated theoretically and experimentally the flow problems with surfactant as the details are being given in the starting. However, the present authors were unable to locate any theoretical or experimental work dealing with the study of particle-in-cell model where the particle is non-porous and is coated with surfactant. Thus there is a need for investigation of such problems hence, the objective of the present work is to investigate, using cell model the problem of slow viscous flow through a swarm of cylindrical particles, where each particle consists of a solid core covered by a liquid shell coated with monomolecular layer of surfactant as shown in Fig. 1. On the hypothetical surface, continuity of radial component of velocity and the boundary conditions of all the four different cell models (Happel, Kuwabara, Kvashin and Mehta - Morse) are used; Boussinesq [1] and Scriven [3] boundary condition is used on the surface of particle which is covered with surfactant. The present problem discusses the flow characteristic when surface of particle is not covered by porous layer instead; it is covered with surfactant. Hence the flow in both the regions are governed by Stokes equation. Drag force experienced by particles in a cell is evaluated. The earlier results reported for drag by Happel, Kuwabara, Kvashin and Mehta–Morse for flow past a solid cylinder is evaluated.

2. Mathematical formulation

Here, we consider a concentrated system (membrane) of identical particles each of radius a, consisting of a solid core of radius $\overline{R}$ covered by a liquid shell of thickness $\overline{\delta}$ coated with a surfactant layer $\overline{r} = a$ with surface shear viscosity $\overline{\epsilon}$ and surface dilatational viscosity $\overline{\kappa}$. The problem will be investigated using the cell method; thus, it is assumed that each particle is located inside a concentric cylindrical cell of radius b (Fig. 1). We assume that the fluid approaches the hypothetical cell surface and passes across the cylinder perpendicular to the axis of the cylinder (z-axis) with velocity U.

Let us introduce a cylindrical co-ordinate system $(\overline{r}, \theta, \overline{z})$ with the origin located at the particle center, $\overline{z}$ axis oriented along the cylinder and the polar axis directed along the flow in the direction of the free stream velocity U approaching the system. Due to axis-symmetry all the physical quantities are independent of $\overline{z}$. Thus we have $\frac{\partial}{\partial \overline{z}} = 0$. The flow inside the shell ($\overline{R} \le \overline{r} \le a$) and outside the shell ($a \le \overline{r} \le b$) are governed by radial and transverse components of Stokes equation and the equation of continuity,

$$\frac{1}{\mu_i}\frac{\partial \overline{p}_i}{\partial \overline{r}} = \frac{\partial^2 \overline{u}_i}{\partial \overline{r}^2} + \frac{1}{\overline{r}^2}\frac{\partial^2 \overline{u}_i}{\partial \theta^2} + \frac{1}{\overline{r}}\frac{\partial \overline{u}_i}{\partial \overline{r}} - \frac{\overline{u}_i}{\overline{r}^2} - \frac{2}{\overline{r}^2}\frac{\partial \overline{v}_i}{\partial \theta} \quad (1)$$

$$\frac{1}{\mu_i \overline{r}}\frac{\partial \overline{p}_i}{\partial \theta} = \frac{\partial^2 \overline{v}_i}{\partial \overline{r}^2} + \frac{1}{\overline{r}^2}\frac{\partial^2 \overline{v}_i}{\partial \theta^2} + \frac{1}{\overline{r}}\frac{\partial \overline{v}_i}{\partial \overline{r}} - \frac{\overline{v}_i}{\overline{r}^2} + \frac{2}{\overline{r}^2}\frac{\partial \overline{u}_i}{\partial \theta} \quad (2)$$