European Journal of Mechanics / B Fluids 69 (2018) 76–86

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

The computation of forward speed Green function in cylindrical coordinate system

Hui Li [a], Li-zhu Hao [a,*], Xiao-bo Chen [b], Hui-long Ren [a]

[a] College of Shipbuilding Engineering, Harbin Engineering University, Harbin, Heilongjiang, 150001, China
[b] Deepwater Technology Research Center, Bureau Veritas, Singapore

A R T I C L E I N F O

Article history:
Received 30 October 2017
Received in revised form 27 January 2018
Accepted 27 January 2018
Available online 3 February 2018

MSC:
32S05
35J08
76B07

Keywords:
Forward speed Green function
Cylindrical coordinate system
Singular point
Wave contours

A B S T R A C T

Recently, a Rankine–Kelvin hybrid method based on meshless cylinder control surface has been developed to solve the ship seakeeping with forward-speed problem. In the external domain, the integration of forward speed Green function needs to be calculated on the cylinder control surface. However, the current computation methods of forward speed Green function are most based on studies in Cartesian coordinate system and not convenient to be integrated on the cylindrical surface. The forward speed Green function and its derivatives are then studied in cylindrical coordinate system in this paper. The double Fourier integral written in tow-fold polar and wavenumber integral is analysed. The polar integral is first performed by using the theorem of residues unlike the classical formulation in which the wavenumber integral was performed before the polar integral. The resultant wavenumber integral is reformulated by removing the weak singularities and evaluated numerically. The numerical results are validated through comparison with those calculated in Cartesian coordinate system, and the wave contours due to a translating pulsating source submerged below the free surface of infinite water depth are given.

1. Introduction

The boundary element method associated with the free-surface Green function which is required to integrate over each discretized panel is conventional approach to solve the hydrodynamic problem. This method works well in some particular cases like wave-body interaction without forward speed [1]. Unfortunately, it fails to be generalized to the problem of the free-surface flow induced by a ship advancing in waves [2–8]. The boundary integral can be reduced to an integral over the body surface and along the waterline, in the limit of smaller and smaller panels (shorter and shorter segment), the integral equation could not produce correct and convergent results [9]. The main reason lies in the complexly singular and highly oscillatory behaviours of the forward speed Green function when both field and source points are in the vicinity of the free surface [10], or waves near the origin of impulsive point are found to diminish continually in length and to increase continually in height.

In order to overcome difficulties associated with the singular and highly oscillatory behaviours of the Green function when solving the forward speed problem, a Rankine–Kelvin hybrid method is proposed, the whole fluid domain is divided into the external domain and internal domain with a smooth control surface, the forward speed Green function is used in the external domain, and the Rankine source is applied in the internal domain. Different from the traditional domain decomposition strategy, the control surface is not panelized in the present Rankine–Kelvin hybrid method, the difficulties mentioned above can be solved by integration of the free surface Green function on the control surface, the velocity potential and its normal derivative on the control surface are expanded into the sum of elementary functions and the relationship between them can be established by solving the boundary integral equation in the external domain. The forward speed hydrodynamic problem can be solved by the boundary integral equation in the internal domain with the continuous conditions cross the control surface.

Ten & Chen [11] adopted a hemi-sphere as the control surface and elementary functions consist of Legendre polynomials and Fourier series, but it is realized that the integral of the Green function on the hemi-sphere control surface is difficult to be calculated in implementation. For the purpose of simplification, an infinite vertical cylinder control surface in deep water is used to keep the independence of horizontal and vertical variables, the velocity potential and its normal derivative are expressed in the form of Fourier–Laguerre series. Hui Li et al. [12], Liang and Chen [13] have successfully applied it to solve the zero speed problem and shown the soundness of the method.

* Corresponding author.
E-mail address: haolizhu@hrbeu.edu.cn (L.-z. Hao).

https://doi.org/10.1016/j.euromechflu.2018.01.005
0997-7546/

When considering a ship advancing in waves, the influence coefficients in the integral equation of external domain are then associated with all elementary Fourier–Laguerre distribution and represented by six-fold integrations including the double integration of Green function (represented by a double Fourier integral) on the cylinder control surface and the double integral following Galerkin collocation. The calculation of influence coefficients is of vital importance. Forward speed Green function in frequency domain is first derived by Haskind [14] which expressed by double Fourier integral, to improve the accuracy and efficiency when solving Haskind source, different forms of Green function are proposed such as Havelock form [15–17] and Michell form [18,19]. Another different forward speed Green function with single integral is Bessho form [20], which is convenient to be calculated and adopted to solve the hydrodynamic problem of a submerged sphere with the steepest-descent method by Iwashita and Ohkusu [21,22], and many other scholars [23–26] have also done much work on the fast integration method. Unfortunately, most previous studies about the forward speed Green function (Havelock form, Michell form, Bessho form) are performed in the Cartesian coordinate system, and this will make it hard to carry out the integration of forward speed Green function on the meshless cylinder control surface. To make the six-fold integrals calculated analytically and reduced to single integrals in wavenumber, Green function (Haskind source) and its derivatives are represented as a new form and computed in cylindrical coordinate system with the polar integral performed first in this paper, this will lay a foundation for the application of Rankine–Kelvin hybrid method with meshless cylinder control surface in forward speed problem.

In this work, firstly, the double Fourier integral of forward speed Green function is studied in cylindrical coordinate system. A new function noted as $g_n(k)$ is introduced to represent the poplar integral. This θ-integral is analytically evaluated in complex plane by using the theorem of residues. There are weak singularities in the resultant wavenumber integral. The singularities of the wavenumber integral are removed by subtracting a singular term which can be evaluated analytically. In addition, the reason of limit $\varepsilon = 0^+$ in forward speed Green function is explained, and the highly oscillatory property about the integrand of the derivatives of Green function is analysed when the field point and source point are on the free surface. At last, the numerical results are compared with those obtained from the Cartesian coordinate system, and excellent agreement is found.

2. Forward speed Green function in cylindrical coordinate system

We define a Cartesian coordinate system $O-xyz$ so that the origin is located on the undisturbed free surface and the axis O_z points downwards, the axis O_x points in the direction of ship advancing with the speed U. We also define a cylindrical coordinate system (ρ, φ, z) and the origin is fixed at the same position with that of the Cartesian coordinate system. These two systems have the following relationship:

$$(x, y, z) = (\rho\cos\varphi, \rho\sin\varphi, z). \tag{1}$$

The free surface boundary condition that forward speed Green function satisfies is as follow

$$-\omega^2 G + 2Ui\omega\frac{\partial}{\partial x}G + U^2\frac{\partial^2}{\partial x^2}G - g\frac{\partial}{\partial z}G = 0 \quad z = 0 \tag{2}$$

where g is gravitational acceleration, ω denotes the frequency of oscillation and the time-harmonic oscillator $e^{-i\omega t}$ is applied and omitted for the sake of simplicity.

In the Cartesian coordinate system, forward speed Green function in frequency domain used here is given in the form of

$$G(P, Q) = G^S + G^F. \tag{3}$$

In which, $G^S = \frac{1}{r(P,Q)} - \frac{1}{r'(P,Q')}$, and $G^F = -\frac{1}{\pi}\lim_{\varepsilon\to 0^+}\int_{-\infty}^{+\infty}\int_{-\pi}^{+\pi}\frac{e^{-k(z+\zeta)-ik[\cos\theta(x-\xi)-\sin\theta(y-\eta)]}}{D+i\varepsilon D_f}kd\theta dk$. The functions D and D_f are defined as $D = (Fkcos\theta - f)^2 - k$, $D_f \equiv \partial D/\partial f = 2(f - Fk\cos\theta)$ and $F = U/\sqrt{gL}$, $f = \omega\sqrt{L/g}$, L denotes the reference length. We should note that the expression of G^F involves $-(y-\eta)$ and $-(z+\zeta)$ instead of $y-\eta$ and $z+\zeta$ as usual, this difference stems from the z axis points down (instead of up as is most often done in the literature).

$\frac{1}{r(P,Q)}$ is the Rankine source defined as that associated with the distance between the field point $P(x, y, z)$ and the source point $Q(\xi, \eta, \zeta)$, and can be written as follow

$$\frac{1}{r(P,Q)} = \frac{1}{\sqrt{R^2 + Z^2}} \tag{4}$$

with $R = \sqrt{(x-\xi)^2 + (y-\eta)^2}$, $Z = |z-\zeta|$.

As

$$\frac{1}{\sqrt{R^2+Z^2}} = \int_0^\infty e^{-kZ}J_0(kR)\,dk. \tag{5}$$

By using the Graf's additional theorem

$$J_0(kR) = \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}J_m(k\rho)J_m(k\rho'). \tag{6}$$

Finally, the Rankine source can be then expressed as:

$$\frac{1}{r(P,Q)} = \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}\int_0^\infty e^{-k|z-\zeta|}J_m(k\rho)J_m(k\rho')\,dk. \tag{7}$$

In a similar way, we have

$$\frac{1}{r'(P,Q')} = \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi')}\int_0^\infty e^{-k(z+\zeta)}J_m(k\rho)J_m(k\rho')\,dk. \tag{8}$$

Based on Eq. (1) and generation function of Bessel function

$$e^{-ik(xcos\theta - ysin\theta)} = e^{-ik\rho cos(\varphi+\theta)} = \sum_{n=-\infty}^{+\infty}(-i)^n J_n(k\rho)\,e^{in(\varphi+\theta)} \tag{9}$$

$$e^{ik(\xi cos\theta - \eta sin\theta)} = e^{ik\rho' cos(\varphi'+\theta)} = \sum_{n=-\infty}^{+\infty} i^n J_n(k\rho')\,e^{-in(\varphi'+\theta)}. \tag{10}$$

Then we can get

$$e^{-ik(xcos\theta - ysin\theta)}e^{ik(\xi cos\theta - \eta sin\theta)} = \sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}(-i)^{p-q}J_p(k\rho)J_q(k\rho') \times e^{i\theta(p-q)}e^{ip\varphi}e^{-iq\varphi'}. \tag{11}$$