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# Analysis of double slip model for a partially filled porous microchannel—An exact solution

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ABSTRACT

Forced convective flow in a microchannel partially filled with a porous medium is analytically modeled with the velocity slips at the solid walls and the porous-fluid interfaces. The inertial effect in the porous medium is taken into account. Explicit solutions for the velocity distribution for a partially filled porous channel are obtained. A flow heterogeneity coefficient based on the flow characteristics of the porous medium is proposed, which is used for evaluating the flow characteristics of a partially filled porous channel. The effects of the Darcy number, porosity, inertial constant, Reynolds number, Knudsen number, and the interfacial velocity slip coefficient on the flow characteristics are analyzed. It is shown that the friction factor decreases with an increase in the Darcy number, and the Knudsen number, and a decrease in the dimensionless porous thickness. The flow heterogeneity coefficient increases with an increase in the dimensionless porous thickness and the Darcy number, and a decrease in $f \cdot Re^2$ and the Knudsen number. The present work is useful for better understanding of boundary and interfacial velocity slip for high velocity flow in a porous-fluid micro-gap. This work also provides an accurate benchmark for various numerical schemes, which involve porous/fluid interactions.



## 1. Introduction

Transport through a porous medium has been studied for many decades due to its wide ranging applications in nature and engineering, such as crude oil extraction [1], soil pollution [2], solar thermal utilization [3], groundwater flow [4], and electronic cooling [5]. Zeng and Grigg [6] performed a study on high velocity flow in a porous medium and presented the criterion for identifying the beginning of the non-Darcy flow. Teng and Zhao [7] presented a dimensionless form of the non-Darcy flow in porous medium with the Brinkman term but without the quadratic term. A detailed investigation of the different flow regimes in porous media is given in Ref. [8]. Wang et al. [9] used the homogenization theory to numerically investigate the flow through porous media. Regulski et al. [10] performed an experimental study on pressure loss in a ceramic foam channel using a Lattice Boltzmann simulation. Sobieski and Zhang [11] presented a multi-scale theoretical study of flow resistance in porous media using the discrete element method and a Forchheimer model.

For a domain, which includes both a porous medium and a clear fluid, several flow coupling conditions at the porous-fluid interface were proposed and analyzed [12]. Beavers and Joseph [13] treated the coupling of fluid flow in a domain with a porous-fluid interface, and presented a velocity jump model at the interface. Vafai and Thiyagaraja [14] presented the first complete analytical treatment of the interface condition between a porous medium and a clear fluid, and demonstrated that the interface condition can be taken care of by using the first principles governing equations and the standard interface matching conditions using matched asymptotic expansions and local volume averaging approach. Vafai and Kim [15] revisited the porous-fluid coupling problem that was treated by Beavers and Joseph. Sahraoui and Kaviany [16] performed a direct numerical simulation of forced convection of the porous/fluid system with Beavers–Joseph interfacial condition with a velocity slip. Xu et al. [17,18] employed the continuity of the velocity and shear stress at the interface, similar to what was done earlier by Vafai and Thiyagaraja [14], to analytically study the forced convection in channels partially filled with metal foams. Alazmi and Vafai [12] analyzed different kinds of velocity coupling conditions at the porous-fluid interface, and compared the velocity profiles with these interfacial coupling conditions. They demonstrated that the velocity profiles predicted with different interfacial conditions are very close to each other. Yang and Vafai [19] performed an analytical study on the forced convection in a parallel-plate channel with a porous medium partially filled at the center.

Research on flow and thermal characteristics of a microchannel is very useful for electronics cooling, which has been analyzed in

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**Nomenclature**

| | |
|---|---|
| $C_E$ | Ergun constant |
| Da | Darcy number |
| $D_e$ | Equivalent diameter, m |
| $K$ | Permeability, $m^2$ |
| $p$ | Pressure, N $m^{-2}$ |
| $q_m$ | Mass flow rate, kg $s^{-1}$ |
| Re | Reynolds number, $\mathrm{Re} = \rho_f u_m \cdot 2H/\mu_f$ |
| $s$ | Shape factor |
| $u_m$ | Mean velocity, m $s^{-1}$ |
| $u$, $v$ | Velocity components respectively at $x$, $y$ directions, m $s^{-1}$ |
| u | Velocity vector, m $s^{-1}$ |
| $U$ | Dimensionless $x$, $y$ velocities |
| $U_{b1}$ | Dimensionless boundary velocity at the bottom wall |
| $U_{b2}$ | Dimensionless interfacial velocity at the upper wall |
| $U_{i1}$ | Dimensionless interfacial velocity at the bottom porous/fluid interface |
| $U_{i2}$ | Dimensionless interfacial velocity at the upper porous/fluid interface |
| $x$, $y$ | Horizontal and vertical positions, m |
| $Y$ | Dimensionless vertical positions |

**Greek symbols**

| | |
|---|---|
| $\alpha^*$ | Interfacial slip constant |
| $\beta_v$ | Boundary slip coefficient of velocity |
| $\beta_i$ | Interfacial slip coefficient of velocity |
| $\varepsilon$ | Porosity |
| $\lambda$ | Molecular mean free path, m |
| $\mu$ | Dynamic viscosity, kg $m^{-1}$ $s^{-1}$ |
| $\xi$ | Flow heterogeneity coefficient |
| $\rho$ | Density, kg $m^{-3}$ |
| $\sigma_v$ | Tangential momentum accommodation coefficient |

**Subscripts**

| | |
|---|---|
| 1 | Bottom plate |
| 2 | Upper plate |
| b | Boundary |
| i | Interface between porous region and fluid region |
| f | Fluid |
| m | Mean |
| p | Porous |

several publications [20–22]. Wang et al. [23] performed an analytical study for forced convective heat transfer in a symmetrical microchannel filled with a porous medium core. For small scale target dimensions, the thermal performance of a microchannel is preferable to that of an impinging jet [20]. Thus, a microchannel combined with porous medium can be an efficient combination that requires further study.

The slip phenomenon for a microchannel with a fluid–solid boundary is the focus of the present study, which exists for small duct scales or a rarefied gas. In the present work, the velocity slip at the fluid–solid boundary and porous-fluid interfaces are taken into account as well as the inertial effect in the porous region. To the best of authors' knowledge, this is the first consideration of these effects in microchannels. The analytical solution for the velocity is obtained and the effects of the key parameters on the flow characteristics are analyzed in detail.

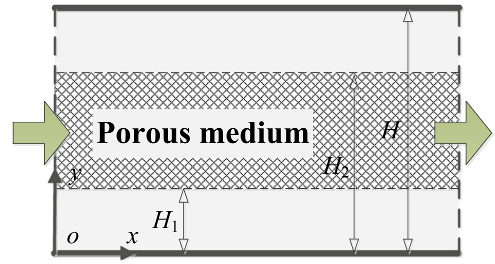


**Fig. 1.** Schematic of the fluid flow in a microchannel with a porous core configuration in the present study.

## 2. Analysis

### 2.1. Physical problem

The problem under consideration is shown in Fig. 1. The parallel-plate microchannel with height H is partially filled with a porous medium at the center and two fluid regions near the wall. The height of the bottom fluid region is $H_1$, and the distance from the upper edge of the porous medium to the bottom plate is $H_2$. Pressure-driven fluid flows through the channel. Due to the existence of the porous medium at the center, the flow characteristics in the porous region is quite different from that of the fluid region. The fluid flow is considered to be fully developed and laminar, and the porous medium is considered to be isotropic. The flow slip at the solid walls along with the inertial effect are considered.

### 2.2. Governing equations

The governing equation for flow through porous media can be written as follows [24]:

$$\frac{\rho_f}{\varepsilon}\left[\frac{\partial \mathrm{u}}{\partial t} + \nabla\left(\frac{\mathrm{u}\cdot\mathrm{u}}{\varepsilon}\right)\right] = -\nabla p + \mu_{eff}\nabla^2\mathrm{u} - \frac{\mu_f}{K}\mathrm{u} - \frac{\rho_f C_E}{\sqrt{K}}|\mathrm{u}|\,\mathrm{u}. \quad (1)$$

The above equation is a complete form of the fluid flow through a porous medium based on the volume averaging method.

Based on the stated assumptions, the simplified equations for flow through a microchannel which is partially filled with a porous medium can be written as:

(1) The bottom fluid region ($0 < y < H_1$ and $H_2 < y < H$):

$$\mu_f \frac{\partial^2 u}{\partial y^2} - \frac{dp}{dx} = 0. \quad (2)$$

(2) The porous medium region ($H_1 \le y \le H_2$):

$$u = u_p,\ -\frac{\mu_f}{K}u - \frac{\rho_f C_E}{\sqrt{K}}u^2 - \frac{dp}{dx} = 0. \quad (3)$$

### 2.3. The closure conditions

Knudsen number is an important dimensionless parameter to evaluate the flow regime. It is expressed as $Kn = \lambda/H$. Fig. 2 shows the schematic diagram of different flow models with the variation of Knudsen number. When the Knudsen number is less than $10^{-3}$, the continuum model can be used. The flow slip at the solid walls is notable when the Knudsen number is greater than $10^{-3}$, as is the case for the convection in a microchannel. The range $10^{-3} < Kn < 10^{-1}$ corresponds to the slip flow regime. The transition flow is in the range $10^{-1} < Kn < 10$ and the free molecule flow is in the range $Kn > 10$. The Navier–Stokes equation can still be applied for modeling fluid flow in the range $10^{-3} < Kn < 10^{-1}$ by involving the slip effect in the boundary conditions. At the top and bottom