European Journal of Mechanics / B Fluids 68 (2018) 39–44

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Film flow thickness along the outer surface of rotating cones

Takahiro Adachi [a,*], Yutaro Takahashi [a], Junnosuke Okajima [b]

[a] *Department of Systems Design Engineering, Akita University, Tegata-Gakuen 1-1, Akita 010-8502, Japan*
[b] *Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan*

ARTICLE INFO

Article history:
Received 28 July 2017
Received in revised form 4 November 2017
Accepted 14 November 2017
Available online 22 November 2017

Keywords:
Rotating cone
Liquid film flow
Film thickness
Boundary layer theory
Rayleigh's criterion

ABSTRACT

A thin liquid film flow goes fully upward along the outer surface of a rotating cone, when the cone is immersed in the liquid, turned upside down and rotated. We derive equations for the velocities and the film thickness of the rising film flow by using the boundary layer theory, and solve them analytically and numerically. The film thickness is obtained analytically in the centrifugal zone, while it is numerically in the Coriolis zone where two different branches with a turning point are found. It is proven that the upper branch is unstable for normalized film thickness $\delta^+ > 3$ from Rayleigh's criterion.

1. Introduction

In the atomization process of a liquid, there are two types of atomizers. One is an atomizer that uses a jet flow, while the other is a centrifugal atomizer that uses the centrifugal force. In the former devices, it is common to use a liquid jet driven through a nozzle at high pressure generated by a compressor. However, the system based on the liquid jet tends to become large because fans, pumps, etc. as well as compressors are necessary for operating it. On the other hand, the latter centrifugal atomizers are quite compact and make it easier to control the characteristics of the atomization such as droplet diameter, quantity of the mist flow composed of the atomized droplets, etc. In the centrifugal atomizers, the droplets are generated by breaking the film flow released from a rotating device or by thinning it not to keep its filmwise condition, where the film thickness as well as the corresponding droplet diameter and the quantity can be adjusted by changing the rotation rate. Therefore, it is important to control the film thickness on the centrifugal atomizers by knowing the relation between the thickness, the rotation rate, etc. for various practical applications.

There are many types of centrifugal atomizers. It is well known, for example, that the liquid rises along the inner surface of a rotating hollow cone due to the centrifugal force. The fluid migrates up the internal wall of the cone under the centrifugal force generating the thin liquid film flow. Bruin [1] has analytically obtained velocity distributions and the film thickness in a liquid film over a rotating conical inner surface. He used the boundary layer theory assuming the film thickness δ is much smaller than the representative radius r_0 such that $r_0 \gg \delta$, where he did not show the representative length scale explicitly. Makarytchev el al.[2] have modified Bruin's model by introducing a better normalization considering the multiplier $\sin\beta$, where β is a half tip angle of cone. They obtained a simpler form of velocity profiles. It should be noted that the results of both Bruin and Makarytchev et al. were the same in a centrifugal zone in which the centrifugal force is dominant, while the results are different from each other in a Coriolis zone in which the Coriolis force is also dominant in addition to the centrifugal force. This is because their normalizations were different from each other. Bruin and Makarytchev et al. obtained the film thickness in the centrifugal zone, but did not explicitly obtain it in the Coriolis zone. This is mainly because the equation for the film thickness in the Coriolis zone is a nonlinear equation and difficult to solve. Furthermore, Makarytchev et al. [3] have studied the structure and regimes of liquid film flow in spinning cone columns. They classified the flow regimes such as the inner inlet-dependent, the intermediate Coriolis, and the outer centrifugal zones using a radial length scale r_0, where they proposed a normalization of the radial scale r_0 at which the thickness of the film becomes equal to the Ekman thickness. Not only for theoretical observation but Makarytchev et al. [4] have also performed an experiment to measure the film thickness. They used a method of the intensity of induced fluorescence illustrated by an ultra violet light source, and obtained the film thickness of wavy liquid film flow on rotating conical inner surfaces, where the film thickness is larger than the thickness in the centrifugal and Coriolis zones. In addition, Symons and Bizard [5] also have performed an experiment to measure the fluid flow thickness within a rotating cone by using an optical

---

* Corresponding author.
E-mail address: adachi@gipc.akita-u.ac.jp (T. Adachi).

https://doi.org/10.1016/j.euromechflu.2017.11.003

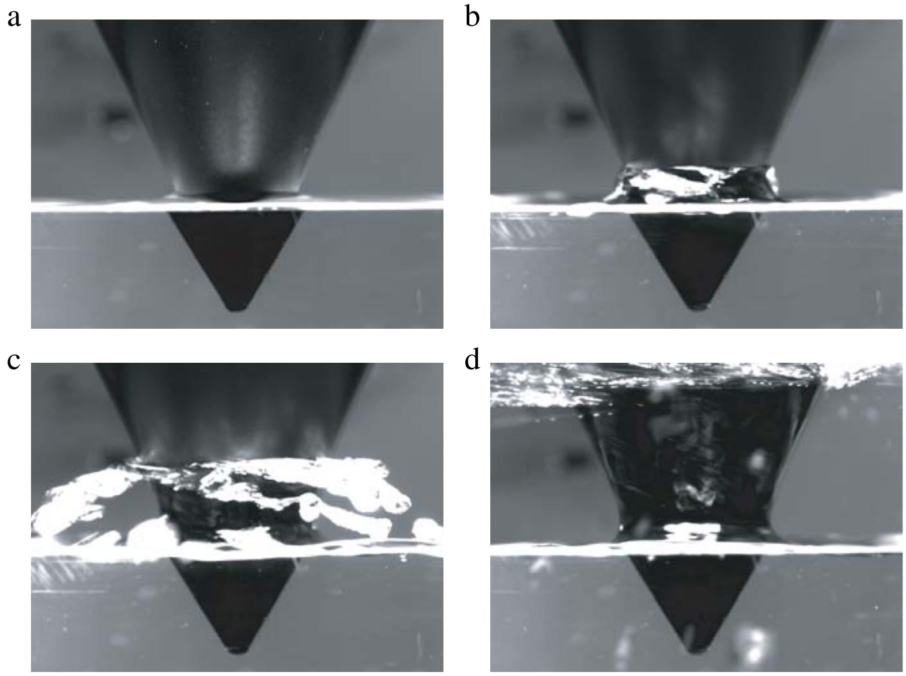

Fig. 1. Visualization photographs of rising film flow.

Nomenclature

E:	Ekman number
Fr:	Froude number
g:	Gravitational acceleration
P:	Pressure
Q_0:	Volumetric flow rate
Q_0^+:	Dimensionless volumetric flow rate
Ro:	Rossby number
r:	Coordinate along the cone surface from apex
r_0:	Distance from apex, representative length scale
s:	Coordinate perpendicular to the cone surface

Greek symbols

β:	Half tip angle of cone
δ:	Film thickness (perpendicular to the cone surface)
δ_E:	Ekman thickness
δ^+:	Dimensionless film thickness
η:	Dimensionless coordinate in the r direction
θ:	Polar angle from the axis in spherical coordinate system
ν:	Kinematic viscosity
ρ:	Density
σ:	Dimensionless coordinate in the s direction
ϕ:	Tangential angle in spherical coordinate system
ω:	Rotation rate of the cone

method. They used a high viscosity fluid and examined the effect of gravity in the centrifugal zone.

Contrary to the film flow within the rotating hollow cone, Adachi et al. [6] have found an unique flow phenomena. Namely, a thin liquid film flow rising along the outer surface of the rotating cone is generated when the cone is immersed and rotates in the liquid by turning the top upside down. Fig. 1 shows the flow visualization of the phenomena with a high-speed video camera, where the rotation rate of the cone is gradually changed from 0 to 6000 rpm. Fig. 1(a) shows an initial state where the cone is in a state of rest. Once the cone begins to rotate, water is deformed and lifted up at the vicinity of the cone surface as seen in Fig. 1(b). However, the water does not go up anymore at that time because the rotation rate is small. When the rotation rate is further increased, the deformation of the water becomes larger as seen in Fig. 1(c), and the water surface winding around the cone rises higher due to the larger degree of the deformation caused by the lifting up. After that, a mass of water is scattered out in radial and tangential directions. Subsequently, a thin liquid film flow is generated and rises along the outer surface on the cone as seen in Fig. 1(d). We call the flow phenomena a pumping-up mechanism caused by the thin liquid film flow. Indeed, it is comprehensible that the liquid rises along the inner surface of a rotating hollow cone due to the centrifugal force but there is only the research of Adachi et al. [6] on the phenomena that the liquid rises along the outer surface of the rotating cone and does not separate from the surface. Recently, Adachi [7] has applied the pumping-up mechanism into oxygen mass transfer and showed that the mist flow composed of atomized water droplets generated by the mechanism is effective for aeration.

In this paper, we focus on the liquid film flow over the outer surface of a rotating cone and derive the equations for the velocity profiles and the film thickness of the rising film flow by using the boundary layer theory. Particular attention is paid to the film thickness. We propose an appropriate nondimensional parameter to express the film flow thickness in both the centrifugal and the Coriolis zones.

2. Mathematical formulation

We consider a laminar steady-state flow around the cone as shown in Fig. 2. Since the water is lifted up in the vicinity of the cone surface when the cone rotates, the water level sinks and the lifting up of the film flow is interrupted. Therefore, in order to generate the rising film flow steadily, we need to supply water. If we supply the water with the flow rate Q_0 shown in Fig. 2(b), the flow rate of the rising film flow balances with the water supply