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Flow and heat due to a surface formed by a vortical source

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ABSTRACT

The existence of exact solutions relevant to the rotating incompressible fluid flows over surfaces formed by the superposition of a uniform source and an irrotational vortex, is described in the present paper. The interest lies in transport phenomena based approach to understanding vortical flows taking place in the vortex flows in geophysics. The vortex structure of the surface is not permitted to persist far away from the surface and the surface is also heated with a finite amount of energy. A treatment of Navier–Stokes equations in cylindrical coordinates shows that such a motion imposes similarity solutions leading to two and three-dimensional flows. Closed-form solutions are proven to be available for zero Reynolds number (Stokes flow) and hence, perturbation solutions follow for small Reynolds numbers. For Reynolds number sufficiently large, numerical solutions clearly point to the formation of a viscous boundary layer near the surface. A continuous decay in the velocity field is anticipated for increasing Reynolds numbers, which, in turn, results in a pronounced thermal layer. The effects of source and vortex strength on the velocity and temperature distributions are eventually investigated. A hotter fluid is found when changing from a source to a sink of the same strength.

1. Introduction

Vortex type structures may appear in various engineering applications whenever a rotary motion is in action. Therefore, the amount of research associated with vortex motion approaches increasingly to an incredible size in the open literature. The present investigation is not directly related with the phenomenon of vortex formation, but it is about the investigation of momentum and thermal layers induced by a moving/rotating wall that is itself formed as a consequence of a point source and a point vortex, which we formally call here as the vortical source.

Vortices are as a result of rotary motion during a physical process, which may be observed in daily life, in many engineering or technological events. For instance, when a fluid is stirred in a cup of tea, or when a boat moves leaving whirlpools at the wake, or most well-known when the wind blows around obstacles (tornado or hurricane), vortices are created, refer to the book by Wu et al. [1] for typical vortex solutions. The study of vortices is very significant since it is necessary to understand the complex phenomena of the momentum, mass and energy transfer which are carried along with the swirling flows. The first study of such kind is attributed to the work of Taylor [2] which investigated the laminar boundary layer swirling flow in a swirl injector or nozzle. For a turbulent swirling flow through a pipe, the decay property of the flow field was later examined by Kreith and Sonju [3]. Thermal features of the boundary layer equation of swirling flow in a convergent nozzle were reported in the numerical work of Back [4]. Heat transfer in a conical hydrocyclone was also computed by Kumari and Nath [5]. Analysis of the swirling flow downstream of a Francis turbine runner was implemented by Susan et al. [6]. Swirling decay and pressure loss across the turbulent boundary layer through a conical swirling chamber were studied by Nouri and Kebriaee [7]. By means of an experimental apparatus of swirl generator, the possible mechanisms of spiral vortex breakdown were recently searched by David et al. [8].

Relevant to the internal flow in turbomachinery, the flow due to a row of vortex and source lines spanning a duct was studied by Falcao and Ferro [9] from which the actuator disc limiting case was also mathematically derived. A series of scientists, such as Serrin [10], Goldshtik and Shtern [11], Shtern et al. [12] and, Chaskalovic and Chauviere [13] devoted their efforts to the mathematical modeling of tornadoes, in an aim to describe some physical features of them from the obtained axisymmetric solutions of the corresponding Navier–Stokes equations. Interaction of a swirling jet with a no-slip surface occurring in combustors, vortex tubes, and tornadoes was highlighted by Shtern and Mi [14]. A new way of perceiving the mature phase of a tornado considering the actions of dust particles was suggested with a newly developed model by Chauviere and Chaskalovic [15]. Interesting analytical vortex solutions to the Navier–Stokes equation were presented by Tryggeson [16]. Recently, other rotating boundary layer flows were investigated for applications such as turbomachinery or

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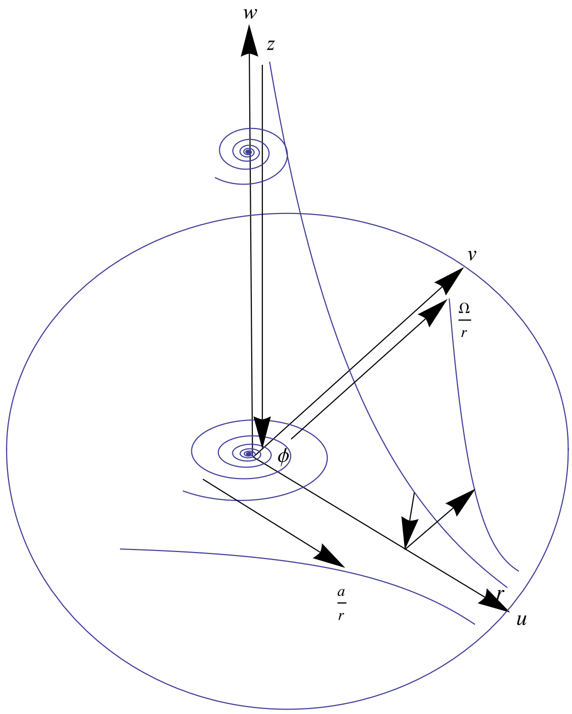

Fig. 1. The set up of physical flow due to a vortical source.

micro-channel heat sinks, refer to Sengupta and Guha [17] and Herrmann-Priesnitz et al. [18].

The present study is essentially different from the available ones in the literature in that the problem under consideration is not the understanding of the vortex formation but, the objective is the study of the flow and heat above the surface generated by a vortical source. To illustrate, such a phenomenon may be observed in geophysical flows, after the burst of a volcano and the ground is fully surrounded by lava radially propagating while swirling like a decaying vortex after an abrupt motion near the volcano, or whirls in the open seas/oceans. Therefore, it is fairly important to know how the momentum and thermal layers will form and possibly evolve into a boundary layer structure of two and three-dimensional type. Here, exact solutions for zero Reynolds number are computed as well as perturbation solutions for small Reynolds numbers. Moderate and large Reynolds numbers flows are numerically simulated. Large Reynolds number analysis openly points to the existence of a boundary layer formation in the vicinity of the wall.

2. Problem formulation

As previously emphasized, we are not interested in the vortex formation, we are rather concerned with the phenomenon of flow of fluid above a vortical source acting like a rigid wall, which may be observed in open seas, in volcanic eruptions or in polymeric manufacturing as depicted in Fig. 1. In place of the traditional vortex formation flow, the flow here is induced by the vortical source of the wall.

It is assumed that there is an abrupt change near the eye of the vortex (z-axis) where the motion is rapid, but the motion tends to rest at far radial distances, behaving like an irrotational vortex. Therefore, a source or sink at the fountain near $r = 0$ is added, so that a fast deformation and rotation is expected there. The motion at $z = 0$ is a combination of first, a point source located at $r = 0$ such that the radial flow is $u = \frac{aR^2}{r}$, where $a < 0$ denotes a sink and $a > 0$ denotes a source, a representing the radial stretching rate (for instance, a spreading lava from the mouth of a volcano). Second, a point vortex exists at $r = 0$ creating a swirling velocity $v = \frac{\Omega R^2}{r}$, with Ω being the core vorticity or vortex swirl rate. Here R is a characteristic vortex radius standing for the vortex core radial extent, refer to [6,16]. Thus, aR^2 is the flow rate per unit axial length through a cylindrical surface at $r = R$ and ΩR^2 represents circulation along the circle with $r = R$ [12]. An unrealistic flow might be created, where flow must adjust in a viscous region. Such a vortex structure is frequently encountered in fluid mechanics; such as the Oseen vortex or the Taylor vortex, see Schlichting [19] and Panton [20]. Refer also to [1,16] for other kind of vortices.

It should be further mentioned that the fluid is incompressible, with constant physical properties, that the steady state flow is sought, and that viscous heating is neglected from the energy equation. The basic steady Navier–Stokes and energy equations of axisymmetric flow rotation in three-dimensional cylindrical coordinates in the usual notations are then

$$\begin{aligned}
&\frac{u}{r} + u_r + w_z = 0,\\
&uu_r - \frac{v^2}{r} + wu_z = -\frac{1}{\rho}p_r + \nu\Big(u_{rr} + \frac{1}{r}u_r - \frac{u}{r^2} + u_{zz}\Big),\\
&uv_r + \frac{uv}{r} + wv_z = \nu\Big(v_{rr} + \frac{1}{r}v_r - \frac{v}{r^2} + v_{zz}\Big),\\
&uw_r + ww_z = -\frac{1}{\rho}p_z + \nu\Big(w_{rr} + \frac{1}{r}w_r + w_{zz}\Big),\\
&uT_r + wT_z = \alpha\Big(T_{rr} + \frac{1}{r}T_r + T_{zz}\Big).
\end{aligned} \tag{2.1}$$

Here (r, ϕ, z) are the cylindrical coordinates, (u, v, w) are the velocity components along (r, ϕ, z) directions, p is the pressure, T is the temperature, ν is the kinematic viscosity, ρ is the density and α is the thermal diffusivity. Although the velocity field has three components (radial, azimuthal and axial), it only depends on two spatial dimensions r and z due to axisymmetry. The pressure gradient in the radial direction is assumed to be negligible as part of the problem definition. Such an exclusion is consistent with the vortex studies in [13,14] as well as the viscous pump of Von Karman flow [21].

Taking into account the vortical source shape of the surface, the accompanied boundary conditions associated with (2.1) are

$$\begin{aligned}
&u(r, z = 0) = \frac{aR^2}{r}, \quad v(r, z = 0) = \frac{\Omega R^2}{r},\\
&w(r, z = 0) = 0, \quad T_w = \frac{T_0}{r} + T_\infty,\\
&u(r, z) \to 0, \quad v(r, z) \to 0,\\
&T(r, z) \to T_\infty \quad \text{as} \quad z \to \infty.
\end{aligned} \tag{2.2}$$

In (2.2), T_w is the wall temperature varying with the radial coordinate and T_∞ is the fixed temperature value of the ambient condition. A heat conduction phenomenon takes place where a finite amount of energy difference T_0/r initially accumulated in the vortical source disperses radially and axially from the source.

3. Similarity solutions

Methodology based on two-dimensional and three-dimensional cases is clearly presented in the sequel making it is easy to follow the line of reasoning.

3.1. Two-dimensional flow case; $\Omega = 0$

When the vortex has a diminishing strength of swirl leaving a source only, i.e., $\Omega = 0$, a two-dimensional flow forms with $v = 0$, only with the radial and axial flow components remaining. Hence,