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The influence of a temperature-dependent interfacial heat release on nonlinear convective oscillations in a two-layer system

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ABSTRACT

Nonlinear convective flows developed under the joint action of buoyant and thermocapillary effects in a two-layer system with rigid heat-insulated lateral walls, have been investigated. The influence of a temperature-dependent interfacial heat release/consumption on oscillatory regimes, has been studied. It is shown that sufficiently strong temperature dependence of interfacial heat sources and heat sinks can lead to the change of the sequence of bifurcations and the development of new nonlinear regimes. Specifically, the period doubling bifurcations have been found.

1. Introduction

Convective phenomena in systems with interfaces have been a subject of an extensive investigation at the past few decades (for a review, see [1,2]). Traditional fields of application of the interfacial convection are chemical engineering [3] and materials processing [4].

It is known that the stability problem for the mechanical equilibrium in a system with an interface is not self-adjoint (see, e.g., [1,5]), thus an oscillatory instability is possible. The hydrodynamic and thermal interaction between convective motions on both sides of the interface can produce oscillations. The oscillatory mechanism of instability was found in [6] for the transformer oil – formic acid system. However, the minimum value of the Grashof number in [6] was achieved on the monotonic mode. Later, some artificial systems with an oscillatory instability have been suggested in [7] (see also [1,5]). A weakly nonlinear theory of oscillations for model systems has been described in [8,9].

In reality, the stability of the mechanical equilibrium is determined by the joint action of the buoyancy effect and the thermocapillary effect. When the system is heated from below, the competition between both effects can lead to the appearance of the specific type of oscillations. This phenomenon was first discovered in [10] (see also [1,11]). Oscillations appearing at the instability threshold have been observed in experiments of Degen et al. (see [12]).

There are various physical phenomena that can be the origin of a heat release on the interface. For example, the interfacial heat release accompanies an interfacial chemical reaction (see, e.g., [13]) and heat consumption accompanies the evaporation [14]. The interfacial heating may be generated, e.g., by an infrared light source. The infrared absorption bands of silicone fluids and water are essentially different [15], therefore the light frequency can be chosen in a way that one of fluids is transparent, while the characteristic length of the light absorption in another liquid is short.

The presence of a constant, spatially uniform heat release at the interface can lead to the appearance of an oscillatory instability [16]. The influence of a constant interfacial heat release and heat consumption on convective oscillations has been studied in the case of *rigid heat-insulated lateral walls*, corresponding to a closed cavity in [17] and in the case of *periodic boundary conditions on the lateral walls*, corresponding to a laterally infinite two-layer system in [18].

The interplay of two independent factors creating the temperature gradient, namely the external heating/cooling from the rigid plate and the interfacial heat release/consumption, significantly influences the basic temperature distribution in a two-layer system, and therefore it can be used for controlling the instabilities. In the absence of the heat source/sink, the temperature gradients in both layers are proportional, and their directions coincide. By means of the heat release/consumption, one can achieve arbitrary directions and absolute values of the temperature gradient in each layer.

In reality, the interfacial heat release/consumption is not constant but is determined by the interfacial temperature. The heat produced on the interface is equal to the heat transported to the

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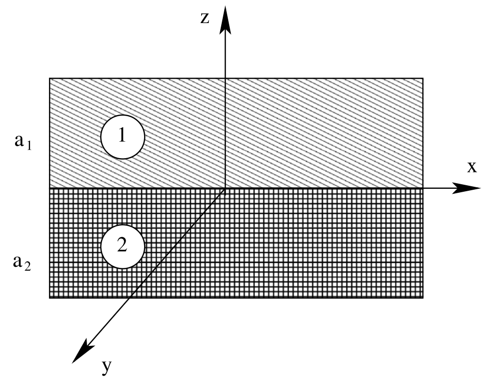

Fig. 1. Geometrical configuration of the two-layer system and coordinate axes.

rigid boundaries of the system. The influence of a temperature-dependent interfacial heat release/consumption on convective regimes in a two-layer system with *periodic boundary conditions on the lateral walls*, has been studied in [19]. The temperature gradients in both layers have been directed *perpendicularly* to the interface.

Let us emphasize that theoretical predictions obtained for the infinite layers cannot be automatically applied for the flows in closed cavities. In the case of periodic boundary conditions one observes waves generated by a convective instability of a parallel flow, while for the observation of waves in a closed cavity a global instability is needed [20]. Also, it should be taken into account, that in the presence of rigid lateral walls the basic flow is not parallel – the lateral walls act as a stationary finite-amplitude perturbation that can produce steady multicellular flow in the part of the cavity and in the whole cavity [20].

In the present paper, we consider the influence of a temperature-dependent interfacial heat release/consumption on nonlinear oscillatory convective regimes in a two-layer system with *rigid heat-insulated lateral walls*. The wide range of parameter Q_T, characterizing the temperature dependence of heat sources and heat sinks at the interface, has been considered. It is shown that sufficiently large values of $|Q_T|$ can lead to the change of the sequence of bifurcations and the development of new nonlinear regimes in the system.

The paper is organized as follows. The mathematical formulation of the problem in a two-layer system is presented in Section 2. The nonlinear approach is described in Section 3. Nonlinear simulations of the finite-amplitude convective regimes are considered in Section 4. Section 5 contains some concluding remarks.

2. Formulation of the problem

We consider a system of two horizontal layers of immiscible viscous fluids with different physical properties (see Fig. 1). The variables referring to the top layer are marked by subscript 1, and the variables referring to the bottom layer are marked by subscript 2. The system is bounded from above and from below by two rigid plates, $z = a_1$ and $z = -a_2$, kept at constant different temperatures (the total temperature drop is θ).

A temperature-dependent interfacial heat release $Q(T_\Gamma)$ is determined as follows:

$$Q(T_\Gamma) = Q(T_\Gamma^0) + Q'(T_\Gamma^0)(T_\Gamma - T_\Gamma^0),$$

where T_Γ is the actual temperature on the interface, and T_Γ^0 is the interfacial temperature at the mechanical equilibrium state. Here $Q(T_\Gamma^0)$ can be positive or negative. Density, kinematic and dynamic viscosity, heat conductivity, thermal diffusivity and heat expansion coefficient of the mth fluid are respectively ρ_m, ν_m, η_m, κ_m, χ_m and β_m; a_m is the thickness of the mth layer ($m = 1, 2$). The interfacial tension σ is a linear function of temperature T: $\sigma = \sigma_0 - \alpha T$, where $\alpha > 0$. We introduce the following notation:

$$\rho = \rho_1/\rho_2,\ \nu = \nu_1/\nu_2,\ \eta = \eta_1/\eta_2,$$

$$\kappa = \kappa_1/\kappa_2,\ \chi = \chi_1/\chi_2,\ \beta = \beta_1/\beta_2,$$

$$a = a_2/a_1.$$

As the units of length, time, velocity, pressure and temperature we use the parameters of the top layer: a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1^2$ and θ, respectively.

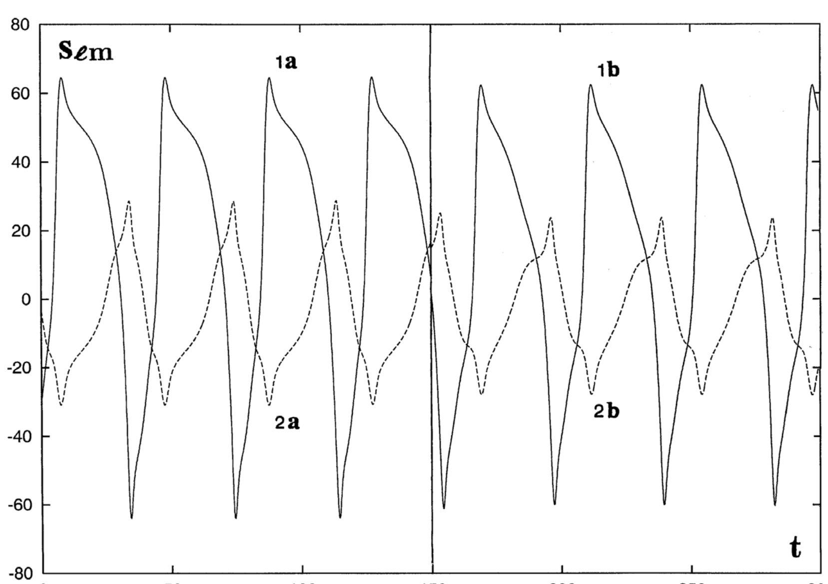

Fig. 2. Dependences of $S_{l,m}$ on time ($m = 1, 2$) at $Q_T = 0$ (lines 1a, 2a); −2.25 (lines 1b, 2b); $G = 162.1$; $K = 0.025$; $G_Q = -25$; $L = 2.74$; $a = 1$.