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Multi-objective economic emission load dispatch problem with trust-region strategy

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ABSTRACT

In this paper, we present a trust region algorithm for solving multi-objective economic emission load dispatch problem (EELD). The trust region algorithm has proven to be a very successful globalization technique for solving a single objective constrained optimization problems. The proposed approach is suitable for multi-objective problem (EELD) such that its objective functions may be ill- defined or having a non convex pareto-optimal front. Also, we identify the weight values which reflect the degree of satisfaction of each objective. The proposed approach is carried out on the standard IEEE 30-bus 6-generator test systems to confirm the effectiveness of the algorithm used to solve the multi-objective problem (EELD). Our results with the proposed approach have been compared to those reported in the literature. The comparison demonstrates the superiority of the proposed approach and confirm its potential to solve the multi-objective problem (EELD).

A Matlab implementation of our algorithm was used in solving one case study and the results are reported.

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1. Introduction

The multi-objective problem (EELD) is of great interest to many researches and several local methods have been proposed to solve it. By local method we mean that the method is designed to converge to optimal solution from closest starting point whether it is local or global one. For a local method, there is no guarantee that it converges if it starts from remote.

The purpose of multi-objective problem (EELD) is to figure out the optimal amount of the generated power of the fossil-based generating units in the system by minimizing the fuel cost and emission level simultaneously, subject to various equality and inequality constraints including the security measures of the power transmission/distribution. Various optimization techniques have been proposed by many researchers to deal with this multi-objective nonlinear programming problem with varying degree of success. In [1,2] the problem has been reduced to a single objective problem by treating the emission as a constraint with a permissible limit. This formulation, however, has severe difficulty in getting the trade-off relation between cost and emission.

Goal programming method was also proposed for the multi-objective problem (EELD) (see [3]). In this method a target or a goal to be achieved for each objective is assigned and the objective function will then try to minimize the distance from the targets to the objectives. Although the method is computationally efficient, it will yield an inferior solution rather than a non-inferior one if the goal point is chosen in the feasible domain.

Heuristic algorithms such as genetic algorithms have been recently proposed for solving multi-objective problem (EELD) (see for example [4–6]). The results reported were promising and encouraging for further research. Moreover the studies on heuristic algorithms over the past few years, have shown that these methods can be efficiently used to eliminate most of difficulties of classical methods (see for example [7–11]). Further more, these methods cannot be used to find pareto-optimal solutions in problems having a non convex pareto-optimal front or ill defined problems.

In this paper, we will use a trust-region globalization strategy to solve the multi-objective problem (EELD). Globalizing strategy means modifying the local method in such a way that it is guaranteed to converge at all even if the starting point is far away from the solution. This approach is applied to solve multi-objective problem (EELD) with no limitation to the number of objective functions and is efficient for solving ill-defined systems and non-convex multi-objective optimization problems.

In this work, we convert the multi-objective problem (EELD) to a single-objective constrained optimization problem by using a weighting approach. The weighting approach is considered as one of the most useful algorithms in treating multi-objective

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optimization problems to generate a wide set of optimal solutions (pareto set) [12]. Also, the effect of changing the weights on cost and emission were studied to show the degree of satisfaction of each objective function.

In this paper, an active set strategy is used together with a multiplier method to convert the single-objective constrained optimization problem to unconstrained optimization problem.

The trust-region strategy for solving the single-objective constrained optimization problem and unconstrained optimization problem has proved to be very successful, both theoretically and practically (see for example [13–18]). Also the trust-region technique for the multi-objective problem has proved to be very successful (see for example [19–21]).

Here, we introduce some notations for subscripted functions denote function values at particular points; for example, $f_k = f(x_k)$, $\nabla_x f_k = \nabla_x f(x_k)$, $L_{k+1} = L(x_{k+1}, \mu_{k+1}, \nu_{k+1})$, $\nabla_x L_k = \nabla_x L(x_k, \mu_k, \nu_k)$, and so on. The matrix H_k denotes the Hessian of the objective function at the point (x_k) or an approximation to it. Finally, all norms are l_2-norms.

This paper is organized as follows: In Section 2 we introduce in details the description of economic emission load dispatch problem. Section 3 is devoted for the mathematical formulation of multi-objective problem (EELD). In Section 4 we give a detailed discussion of the trust region algorithm problem (EELD). Furthermore, we then discuss in detail the implementation of the proposed approach in Section 5. The results and discussions are presented in Section 6. Finally, the conclusion and future works are given in Section 7.

2. Economic emission load dispatch problem

The economic emission load dispatch involves the simultaneous optimization of fuel cost and emission objectives which are conflicting ones. The deterministic problem is formulated as follows:

2.1. Objective functions

There are two objective functions which are described in details as follows:

(1) **Fuel cost objective function**. The classical economic dispatch problem of finding the optimal combination of power generation, which minimize the total fuel cost while satisfying the total required demand can be mathematically stated in [22] as follows:

$$f_1 = \sum_{i=1}^{n} C_i(P_{Gi}) = \sum_{i=1}^{n} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \tag{2.1}$$

where f_1 is the total fuel cost($/h), C_i is the fuel cost of generator i, P_{Gi} is the power generated by generator i, n is the number of generator, and a_i, b_i, c_i, are the fuel cost coefficients of generator i.

(2) **Emission objective function**. The emission function can be expressed as the sum of all types of emission considered as NO_2, SO_2, thermal emission, etc., with suitable pricing or weighting on each pollutant emitted. In the present study, only one type of emission NO_2 is given as a function of generator output, that is, the sum of a quadratic and exponential function:

$$f_2 = \sum_{i=1}^{n} [10^{-2}(\tilde{a}_i + \tilde{b}_i P_{Gi} + \tilde{c}_i P_{Gi}^2) + \xi_i e^{\lambda_i P_{Gi}}], \tag{2.2}$$

where f_2 is the amount of NO_2 emission (ton/h) and $\tilde{a}_i, \tilde{b}_i, \tilde{c}_i, \xi_i$, and λ_i are the coefficients of the ith generator's NO_2 emission characteristic.

2.2. Constraints

The optimization problem is bounded by the following constraints:

(1) **Power balance constraint**. The total power generated must supply the total load demand and the transmission losses

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_{Loss} = 0, \tag{2.3}$$

where P_D is a total load demand, and P_{Loss} represents a transmission losses. The transmission losses are given by [23] as follows:

$$P_{Loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} [A_{ij}(P_i P_j + Q_i Q_j) + B_{ij}(Q_i P_j - P_i Q_j)], \tag{2.4}$$

where $P_i = P_{Gi} - P_{Di}$, $Q_i = Q_{Gi} - Q_{Di}$, $A_{ij} = (R_{ij}/V_i V_j)\cos(\delta_i - \delta_j)$, and $B_{ij} = (R_{ij}/V_i V_j)\sin(\delta_i - \delta_j)$ such that n is a number of buses, R_{ij} is a series resistance connecting buses i and j, P_i is a real power injection at bus i, Q_i a reactive power injection at bus i, V_i is a voltage magnitude at bus i, δ_i is a voltage angle at bus i.

2 **Maximum and minimum limits of power generation**. The power generated P_{Gi} by each generator is constrained between its minimum and maximum limits, i.e., $P_{Gi_{min}} \le P_{Gi} \le P_{Gi_{max}}$, $Q_{Gi_{min}} \le Q_{Gi} \le Q_{Gi_{max}}$, and $V_{i_{min}} \le V_i \le V_{i_{max}}$.

3 **Security constraints**. A mathematical formulation of the security constrained EELD problem would require a very large number of constraints to be considered. However, for typical systems the large proportion of lines has a rather small possibility of becoming overloaded. The EELD problem should consider only the small proportion of lines in violation, or near violation of their respective security limits which are identified as the critical lines. We consider only the critical lines that are binding in the optimal solution. The detection of the critical lines is assumed done by the experiences of the DM. An improvement in the security can be obtained by minimizing the following objective function,

$$S = \sum_{q=1}^{m} \left(\frac{(T_q(P_G))^2}{T_q^{max}} \right),$$

where $T_q(P_G)$ is the real power flow, T_q^{max} is the maximum limit of the real power flow of the qth line, and m is the number of monitored lines. The line flow of the qth line is expressed in terms of the control variables P_{Gs} by utilizing the generalized generation distribution factors (GGDF) in [24] and is given as follows

$$T_q(P_G) = \sum_{i=1}^{n} (D_{qi} P_{Gi}),$$

where D_{qi} is the generalized GGDF for line q due to generator i.

For secure operation, the transmission line loading S_l is restricted by its upper limit as

$$S_l \le S_{lmax}, \quad l = 1, \ldots, n_l,$$

where n_l is the number of transmission line.