European Journal of Mechanics / B Fluids 67 (2018) 366–376

Contents lists available at ScienceDirect

European Journal of Mechanics / B Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Stability of an air-assisted viscous liquid sheet in the presence of acoustic oscillations

Li-jun Yang [a,*], Bo-qi Jia [a], Qing-fei Fu [a], Qi Yang [b]

[a] *School of Astronautics, Beijing University of Aeronautics and Astronautics, Beijing 100191, China*
[b] *Shaanxi Electrical Equipment Institute, Xi'an, 710000, China*

ARTICLE INFO

Article history:
Received 30 June 2017
Received in revised form 25 September 2017
Accepted 2 October 2017
Available online 23 October 2017

Keywords:
Acoustic oscillations
Temporal instability
Viscous potential flow

ABSTRACT

The linear temporal stability of a viscous liquid sheet is studied in the presence of acoustic oscillations. The viscous potential flow theory is applied to account for liquid viscosity. Acoustic oscillations are provided by imposing a sinusoidal oscillation of the gas velocity or density. Results suggest that the viscosity has a stabilizing effect with a zero mean velocity, and dual effects with a non-zero mean velocity. The effect of oscillations at low velocity is more significant than effects realized at high velocity. Oscillations are a destabilizing factor, although they have a weaker effect at a larger frequency than that at a lower frequency due to the liquid viscosity. Acoustic oscillations promote the instability of the liquid sheet; however, the effects of mean velocity, the gas-to-liquid density ratio, liquid sheet thickness and surface tension are analogous, whether acoustic oscillations exist or not.

1. Introduction

Research to date has focused on the breakup of a planar liquid sheet surrounded by a gaseous ambient, providing a theoretical reference for numerous practical applications, such as combustion, gas turbines, diesel engines, spray cooling, surface coating, and atomization. The stability of a moving planar liquid sheet is of great importance to the reliability of engine performance relative to the rupture and atomization of liquid sheets. A sound knowledge of the mechanism of the stability is not only of scientific value, but it is also essential for the design and operation of practical systems.

Hydrodynamic forces are the main cause of disturbance growth in a planar liquid sheet, leading to its ultimate breakup; these effects are generally expressed by Kelvin–Helmholtz (K–H) instability. For this reason, extensive theoretical and experimental investigations have been performed on the K–H instability of planar liquid sheets.

Squire [1] analyzed the linear stability of an inviscid moving liquid sheet surrounded by inviscid gas. His experimental and theoretical results showed that surface tension has a stabilizing effect on the disturbances of the liquid sheet; and the most unstable wave has been found. Dombrowski and Johns [2] examined a viscous liquid sheet in the presence of inviscid gas and predicted the diameter of the liquid droplets formed by the rupture of the liquid film generated by fan-spray nozzles. Crapper et al. [3] accounted for the viscosity of both liquid and gas phases; their results revealed that liquid viscosity has no effect on the initial growth of the sinuous wave and that liquid viscosity could extend the instability frequency range. Li and Tankin [4] demonstrated the instability of a thin moving viscous liquid sheet in a stationary inviscid gas medium. They found two regimes of instability for viscous liquid sheets: the aerodynamic and the viscosity-enhanced instability. For sinuous disturbances, liquid viscosity enhances instability at small Weber numbers, while reducing the growth rate and the dominative wavenumber at large Weber numbers.

In the theoretical works cited above, linear stability analysis was employed for air-assisted liquid sheets without acoustic oscillations. However, pressure pulsations caused by combustion instability generally exist in liquid fuel engines, leading to an oscillating gaseous ambient. The instability of the liquid sheet in an oscillating gaseous ambient is due to a coupling effect of hydrodynamic forces and oscillations. As mentioned above, effects of hydrodynamic forces can be regarded as the K–H instability. Effects of oscillations can also be dealt with using the theory of the parametric instability. Hence the present problem is a combination of two kinds of unstable mechanism: K-H instability and parametric instability.

There are many studies that focus on parametric instability. Faraday [5] first examined the instability of periodic basic flow, which was initially solved theoretically by Benjamin and Ursell [6]. They used the Floquet theory to solve the dispersion equations with a form of Mathieu equations; they explained the unstable and stable regions of the disturbance of the liquid, corresponding to the unstable and stable solutions of Mathieu equations. Their results explained the different experimental phenomena between Faraday, Rayleigh, and Matthiessen.

* Corresponding author.
E-mail address: yanglijun@buaa.edu.cn (L.-j. Yang).

https://doi.org/10.1016/j.euromechflu.2017.10.002
0997-7546/

Kumar et al. [7–9] conducted extensive studies on the parametric instability of liquids, taking viscosity and the Coriolis force into consideration. Kumar [10] presented a study on the parametric instability of a Newtonian and a viscoelastic liquid sheet; and the results were similar to the classic Faraday instability. Rajchenbach et al. [11,12] conducted a procedure that theoretically explained the experimental results. It was found that the excited mode is not necessarily the standing wave that corresponds to the container eigenmode with the frequency closest to $n\Omega/2$, but oscillates exactly at $n\Omega/2$ with the wavenumber given by the dispersion relation, including forcing and dissipation.

For the coupling effect of the K-H instability and the parametric instability, [13] carried out theoretical research on the stability of an unsteady K-H flow, a coupling effect of the K-H instability and the parametric instability in fact. His results showed that the oscillations of the basic flow can cause a subharmonic resonance; so a wave becomes unstable when the wave is neutrally stable, in the absence of the oscillations, and its frequency is half the frequency of the flow oscillations.

In 1977 a linear stability analysis was employed by Dityakin et al. [14] to investigate the instability of an air-assisted inviscid liquid sheet in the presence of acoustic oscillations. Their results showed that the characteristics of the instability of liquid sheets are essentially affected by the velocity pulse of gas. The stable regions and the unstable regions locate discretely. Within the range of Weber number studied, the dominative wavenumber for sinuous mode is identical with that for the varicose mode.

Vorob'Ev [15] examined the instability of inviscid liquid sheets in another inviscid liquid medium with acoustic oscillations. The motion was divided into fast and slow, with allowance for nonlinear acoustic oscillations. The results revealed that the action of the inviscid liquid sheets was determined both by the physical parameters and by the relative position of the liquid sheet to the node and antinode of the acoustic field. Sivadas et al. [16,17] conducted experimental research to observe the effect of acoustic oscillations on air-assisted liquid sheets. They analyzed disintegration characteristics and compared the characteristics with the flow features in the absence of acoustic oscillations. Their results revealed that properly external acoustic oscillations may accelerate the rupture of liquids. Mulmule et al. [18–20] studied the effect of sinusoidal acoustic oscillations on the breakup of the inviscid liquid sheet both theoretically and experimentally. They found that the critical wavelength tends to be smaller, and the size of the droplets formed by the disintegration of the liquid sheet reduces in the presence of acoustic oscillations.

Unfortunately, there are few theoretical studies that consider the coupling effect of K-H instability and the parametric instability for a viscous liquid sheet when there is a non-zero relative velocity between the liquid sheet and oscillating gaseous ambient. For purely K-H instability, the basic flow is steady; hence, the disturbance can be expressed in terms of a normal mode. For purely parametric instability, the Floquet theory can be adopted to describe the disturbance. Thus, both cases have the same solution for the full viscous flow. However, both the purely Floquet theory and normal mode are invalid for the coupling effect of the K-H and the parametric instability; so it is difficult to obtain the exact solution for viscous fluid under this condition. A series of simplifications is essential to solve the problem.

It is well known that the Navier–Stokes equations are satisfied by potential flow, and the viscous term is identically zero when the vorticity is zero, but the viscous stresses are not zero [21]. The differences between the results predicted by the viscous flow and those given by the exact viscous theory are fairly minor in that shear stress is negligible. Joseph et al. [22,23] constructed the viscous potential flow models for both Rayleigh–Taylor instability and the K–H instability problems. Their results illustrated that maximum growth rate, dominative wavenumber, and critical wavelength (which the viscous potential flow theory presents) have only minor errors compared to those given by the same viscous theory. Viscosity also has a significant effect on the instability of liquid sheets. Because of the success of viscous potential flow in the analysis of Rayleigh–Taylor and K-H instability, the linear stability theory and the viscous potential flow theory have been combined here to investigate the instability of the air-assisted liquid sheet in the presence of acoustic oscillations. From an analytical point of view, this paper discusses the effects of the acoustic oscillations and the physical parameters of viscous liquid sheets on the maximum temporal growth rate and the dominant wavenumber.

2. Governing equations

Fig. 1 shows the schematic diagram of a two-dimensional air-assisted liquid sheet with a uniform thickness $2a$, surface tension σ, and dynamic viscosity μ. For the convenience of derivation, the problem is set in a coordinate system moving with the liquid sheet, which is surrounded by a gaseous flow with a velocity U. The coordinate system is chosen so that the x axis is parallel to the direction of the basic flow and the y axis is normal to the unperturbed surface of the liquid sheet, with the origin located in the middle of the nozzle exit. Assumptions are made to suppress the nonlinear terms in the momentum equations: The gas is assumed to be inviscid, the liquid incompressible, and the densities of liquid and gas are ρ_l and ρ_g, respectively. The effect of gravity is neglected.

In this work, acoustic oscillations are expressed by the oscillations of gas velocity or density, only the oscillation amplitude is a function of time. In the presence of acoustic oscillations in the range of 13–3 $\times$ 10^6 Hz [14], it can be assumed that gas velocity and density are constant in an infinitely short time, but vary in a finite time. Therefore, the linear stability theory can be employed for infinitely small time Δt, and the gas velocity and density vary in finite time t, similar to the technique of Dityakin et al. [14]. Because of the complexity of acoustic oscillations, only sinusoidal oscillations of gas velocity and density are studied in this work

When disturbances begin, upper and lower interfaces are displaced and regarded to be one of the following forms:

$$\begin{aligned} &y = (-1)^j a + \eta(x,t), && \text{for sinuous mode} \\ &y = (-1)^j a + (-1)^j \eta(x,t), && \text{for varicose mode} \\ &\eta(x,t) = D(t)\exp(ikx) && \end{aligned} \tag{1}$$

where $j = 0$ and $j = 1$ represent the upper and the lower surfaces, respectively. $k = 2\pi/\lambda$ is the wavenumber of the surface wave, with λ being the wave length. The entire flow field is correspondingly disturbed and deviates from the base (undisturbed) flow described above.

According to Funada et al. [22], the normal stress is an extensional rather than a shear stress, and it is activated by waves on the liquid; the waves are induced more by pressure than by shear. Therefore, the neglect of shear could be reasonable in wave motions, whose viscous resistance is not negligible; this is the situation which may be approximated well by viscous potential flow. In addition, according to Joseph et al. [21], if the momentum equation holds for potential flow, the following condition must be satisfied:

$$\nabla \times (\nabla \cdot \boldsymbol{S}) = 0$$

That is, there exists a real function ς such that:

$$\nabla \cdot \boldsymbol{S} = \nabla \varsigma$$

where $\boldsymbol{S}$ is extra stress given by the fluid constitutive equation; ς is a real function, and $\varsigma = 0$ is suitable for Newtonian fluids of