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Stability of an air–water flow in a semispherical container

Luis Carrión [a,b], Miguel A. Herrada [b,*], Vladimir N. Shtern [c]

[a] *Universidad de las Fuerzas Armadas-ESPE, Av. Gral. Rumiñahui s/n Sangolquí 171103, Ecuador*
[b] *E.S.I, Universidad de Sevilla, Camino de los Descubrimientos s/n 41092, Spain*
[c] *Shtern Research and Consulting, Houston, TX 77096, USA*

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ABSTRACT

This numerical study analyzes the stability of a steady axisymmetric air–water flow driven by a rotating top disk in a sealed semispherical container. A motivation is possible applications in aerial bioreactors. The centrifugal force pushes the air to periphery near the disk, downward near sidewall, toward the axis near the interface, and upward near the axis. This meridional circulation of air drives the water counter-circulation while the centrifugal force tends to induce the water co-circulation. Their competition results in the development of a three-eddy pattern as the rotation intensifies. The air circulation and the water co-circulation are separated by a thin layer of water counter-circulation. It is shown that the time-oscillatory helical instability emerges when the three-eddy pattern is well formed. The azimuthal wave number is $m = 1$ in the shallow-water case and $m = 2$ otherwise. The analysis of flow patterns and critical-disturbance energy distributions indicates that the instability emerges in the air domain and likely is of the shear-layer type.

1. Introduction

An intriguing and important fluid-mechanics phenomenon is the emergence of a local circulation cell in a swirling flow, often referred to as vortex breakdown (VB). VB applications include delta-wing aircraft, where VB is dangerous causing an abrupt change in lift and drag, combustion chambers, where VB is beneficial stabilizing flame, and natural swirling jets like tornadoes, where VB decreases the twister strength. Escudier [1] performed a comprehensive review of early VB studies. More recent works, including VB control strategies, are discussed in Ref. [2].

Vogel [3] and Escudier [4] initiated fundamental VB studies in a sealed cylindrical container with one end disk rotating. An advantage is the closed domain with well-defined and controlled boundary conditions allowing for meaningful comparisons of experimental and numerical results. They well agree as was first shown by Lopez [5]. The analysis of the Vogel–Escudier flow helps understand the VB nature. A recent view is that VB develops via the swirl-decay mechanism [6,7].

While single-fluid VB flows have been studied rather in detail, two-fluid VB flows have not attracted much attention until recent time. The situation changed with the development of aerial bioreactors where air–water flows are used for the growth of tissue culture [8]. The air flow transports the oxygen, required for tissue growth, to the interface and the water circulation enhances mixing of the dissolved oxygen with other ingredients. The tissue fraction is small compared with that of water and is neglected in the studies of flow patterns. The bioreactor applications stimulated the experimental investigations by Lo Jacono et al. [9] and the numerical simulations by Liow et al. [10,11]. Early numerical studies modeled the gas–liquid interface as a symmetry plane [12] and as a deformable stress-free surface [13]. The first work, which is free from these idealizations of the interface, was performed by Brady et al. [14,15].

Further research has revealed that two-fluid VB flows have a number of interesting features absent in single-fluid flows. One striking feature, observed also in the current study, is the existence of a thin circulation layer (TCL) adjacent to the interface. A TCL attached to the entire interface from below was found in water-spout flows occurring in cylindrical [16] and semispherical [17] containers.

Another striking feature is the emergence of an off-axis VB ring in the depth of a lower fluid away from the interface, axis, and walls [18]. Since eddies arise in both fluids, their variety is rich and transformations are numerous. For example, eighteen topological metamorphoses follow each other as the water volume fraction increases in a truncated conical container where a creeping air–water flow is induced by the slowly rotating top disk [19]. The diversity of flow cells and their metamorphoses is even more enriched by the Moffatt eddies [20], which develop near intersections of the end and side walls [21].

* Corresponding author.
E-mail address: herrada@us.es (M.A. Herrada).

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The resulting rather complicated topology raises a question about the flow stability. The stability of one-fluid VB flows in a cylindrical container has been investigated rather in detail. The numerical study of Gelfgat et al. [22,23] showed that the Vogel–Escudier flow can become unstable at either smaller or larger Re than that, at which VB emerges, depending on the length-to-radius ratio, H. The experimental and numerical studies by Escudier [4] and Sorensen et al. [24–26] documented that, as Re increases, the steady axisymmetric VB bubble first develops for $H < 3.2$. For larger H, the flow first becomes unstable with respect to 3D time-oscillatory disturbances with $m = 3$ for $3.2 < H < 4.3$, $m = 2$ for $4.3 < H < 5.2$, and $m = 4$ for $5.2 < H < 5.5$; m is the azimuthal wave number. Herrada et al. [7] found that this instability is of the shear-layer type developing for $H > 5.5$ as well. Unsteady three-dimensional flows resulting from the instability were studied in Refs. [27–29].

Here we numerically investigate the stability of air–water flow in a semispherical container studied by Balci et al. [17]. This flow seems the most appropriate for bioreactor applications since the number of eddies, which can damage the tissue, is minimal here. In the cylindrical bioreactors [9–12], a set of eddies is located near the sidewall-bottom intersection [16]. This eddies are absent in a semispherical container. One more advantage is that semispherical geometry enhances the streamline convergence toward the axis. This convergence strengthens swirl and therefore the centrifugal force, which drives the meridional circulation. Thus, the near-stagnant corner region is removed and the global circulation of ingredients is enhanced.

There is a technical difficulty of studying the stability of a two-fluid flow: the linearization of a rather complicated relation describing the balance of normal stresses at the bent interface. To overcome this problem, an efficient routine was elaborated [30], which in addition facilitates numerical simulations. The routine includes (i) mappings, converting the time-dependent upper and lower fluid regions onto fixed squared domains, (ii) a symbolic toolbox to calculate the analytical Jacobians, and (iii) the Chebyshev grid in both radial and axial directions. Herrada & Montanero proved the method efficiency in their study of liquid-bridge dynamics [30]. Here this numerical technique, being modified and applied for the hemispherical problem, helps investigate and understand the instability nature.

In the rest of this paper, we formulate the problem in Section 2, describe the numerical technique in Section 3, explore the flow stability at the water height h_w, being 0.8, 06, 0.4, and 0.2 of the disk radius R, in Section 4, summarize the results in Section 5, and verify their grid-independence in Appendix.

2. Problem formulation

2.1. Flow geometry

Fig. 1 is a problem schematic. The lower part, $0 < z < h_w$, of the semispherical container is filled with water, the upper part, $h_w < z < R$, is filled with air; r, ϕ, and z are cylindrical coordinates; and $\boldsymbol{g}$ is the gravitational acceleration. The interface is depicted by the thin horizontal line, $z = h_w$. The semispherical wall is stationary. The disk lid, located at $z = R$, rotates with angular velocity Ω; R is the disk and hemisphere radius, which serves as a length scale. The dimensionless control parameters are the water fraction, characterized by the water height $H_w = h_w/R$, and the Reynolds number, $Re = \Omega R^2/\nu_w$, characterizing the rotation strength; ν_w is the kinematic viscosity of water. One our goal is to explore (i) the development of flow instability, as Re increases at a fixed H_w, and (ii) how the critical parameters depend on the water fraction. To this end, we consider $H_w = 0.2$, 0.4, 0.6, and 0.8.

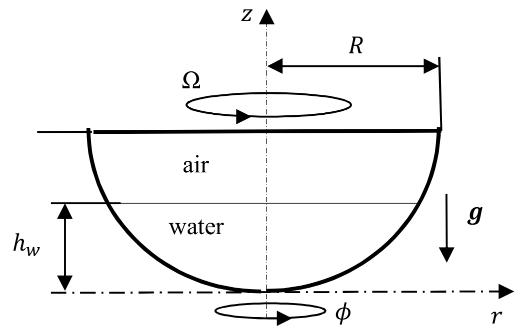

Fig. 1. Schematic of the problem. The lid only rotates.

2.2. Governing equations

Using R, $1/\Omega$, ΩR, and $\rho_w \Omega^2 R^2$ as scales for length, time, velocity, and pressure, respectively, renders all variables dimensionless. We consider a flow of two viscous incompressible immiscible fluids governed by the Navier–Stokes equations,

$$\frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \phi} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\rho_n \frac{\partial p}{\partial r} + \frac{\nu_n}{Re}\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \phi}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \phi} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = -\frac{\rho_n}{r}\frac{\partial p}{\partial \phi} + \frac{\nu_n}{Re}\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \phi}\right), \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \phi} + w\frac{\partial w}{\partial z} = -\rho_n \frac{\partial p}{\partial z} + \frac{\nu_n}{Re}\nabla^2 w, \tag{4}$$

where $\nabla^2 \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator for a scalar field, (u, v, w) are the velocity components in cylindrical coordinates (r, ϕ, z), t is time, and p is pressure. The coefficients, ρ_n and ν_n, are both equal 1 at $n = 1$ (in the water) while $\rho_n = \rho_w/\rho_a$ and $\nu_n = \nu_a/\nu_w$ at $n = 2$ (in the air).

We denote the list (u, v, w, p) as $\boldsymbol{V}$, and look for a solution of the system (1)–(4) in the form

$$\boldsymbol{V} = \boldsymbol{V}_b(r, z) + \varepsilon \boldsymbol{V}_d(r, z)\, e^{(im\phi - i\omega t)} + c.c., \tag{5}$$

where subscripts "b" and "d" denote the base flow and a disturbance, respectively; $c.c.$ denotes the complex conjugate of the preceding term; $\varepsilon \ll 1$ is an amplitude; integer m is an azimuthal wave number; and $\omega = \omega_r + i\omega_i$ is a complex number to be found, with frequency ω_r and growth rate of disturbance ω_i. For a decaying (growing) disturbance, ω_i is negative (positive). The equations governing the base flow result from substituting (5) in system (1)–(4) and setting $\varepsilon = 0$. The terms of order $O(\varepsilon)$ constitute equations governing infinitesimal disturbances.

2.3. Boundary conditions

Eqs. (1)–(4) are solved under the following boundary conditions:

(i) Regularity at the axis, $0 < z < R$, $r = 0$:

(a) $u = v = 0$, $\partial w/\partial r = 0$ (basic flow and $m = 0$ disturbances),
(b) $w_d = 0$, $u_d + m v_d = 0$, $\partial u_d/\partial r = 0$ ($m = 1$ disturbances)
(c) $w_d = u_d = v_d = 0$ ($m > 1$ disturbances)