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A pressure impulse theory for hemispherical liquid impact problems

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ABSTRACT

Liquid impact problems for hemispherical fluid domain are considered. By using the concept of pressure impulse we show that the solution of the flow induced by the impact is reduced to the derivation of Laplace's equation in spherical coordinates with Dirichlet and Neumann boundary conditions. The structure of the flow at the impact moment is deduced from the spherical harmonics representation of the solution. In particular we show that the slip velocity has a logarithmic singularity at the contact line. The theoretical predictions are in very good agreement both qualitatively and quantitatively with the first time step of a numerical simulation with a Navier–Stokes solver named *Gerris*.

1. Introduction

Impacts are extremely brief and violent phenomena involving solids or fluids occurring in very diverse situations: impact of a wave on a seawall [1], the landing of seaplanes [2] and more generally the water entry of solid objects [3,4] or the impact of a liquid drop onto a solid surface [5,6]. As suggested by the variety of previous examples, impact problems involve any geometry and could be defined as a general problem with liquid and solid domains of arbitrary shape (see Fig. 1).

The impulsive aspect of impacts is a common characteristic shared by all these problems. This feature could be defined as a considerable acceleration of a boundary of the system over a very short time. Consequently impact phenomena are unsteady, non-linear and could produce large deformations as in the case of problems involving free-surfaces *e.g.* the run across a river of the Jesus-Christ lizard [7] or the game of stone-skipping [8]. This last problem was applied to naval artillery and studied theoretically by E. de Jonquières [9] in order to explain why the bouncing of cannonball across water improves the range of the shoot.

In this paper we will focus on impact problems involving drops with the emphasis on its impulsive aspect. More specifically we will consider the particular case of hemispherical fluid domains. One example of problem worth of interest in this kind of configuration is the study of the dynamics of a drop sitting on a solid substrate when this last is impacted from beneath (see Fig. 2 left). There are several interesting questions associated to this problem such as: *(i)* What is the minimal impact intensity necessary to eject partially or totally the drop from the solid surface? *(ii)* What are the deformation modes induced by the impact? *(iii)* What is the influence of the substrate's inclination? The two first problematics have already been treated for sessile drops sitting on an oscillating solid substrate [10,11]. The solution of the whole problem as defined here is an ambitious program clearly beyond the scope of this paper. Then we propose to solve a slightly different problem with the same impulsive characteristic. Henceforth we consider a drop of radius R sitting on a larger cylindrical substrate. The substrate is initially risen impulsively toward the drop with a velocity U (see Fig. 2 right). This new configuration, based on the impulsive motion of a solid boundary has already been considered in different contexts [12,13].

The impulsive problem we propose here could also be seen as an impact problem. In fact it is possible to study it experimentally by using a cylinder, assumed nondeformable, with a drop disposed at its top and falling in free fall. When the cylinder hits the ground, the impact imposes a velocity U of the substrate in the reference frame of the drop. Hence we can consider that these points of view are both equivalent. In terms of impulsive impact of liquid bodies on plane wall, a similar problem was studied for different geometries by Tyvand et al. [14] and for cylindrical fluid bodies by Hjelmervik et al. [15]. The aim of the present paper is to study impact problems for a drop disposed onto a cylindrical substrate by using the analogy with the impulsive problem depicted here and with a focus on the flow at the impact moment. In Section 2 we introduce the theoretical framework of the problem, based on the concept of pressure impulse and associated to the impulsive nature of impacts. We deduce that the problem is reduced to the solution of Laplace's equation with Dirichlet and Neumann boundary conditions. In Section 3 we determine the pressure impulse along the wetted region

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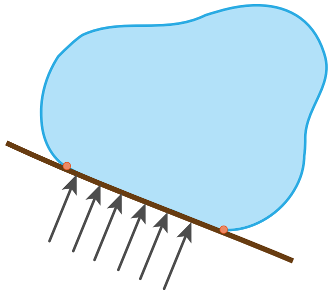

Fig. 1. Sketch of an impact problem involving a liquid domain of arbitrary shape and a solid boundary. The red dots represent the position of the contact line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and the slip velocity by using a spherical harmonics representation of the solution. Then, theoretical predictions are qualitatively and quantitatively compared with numerical solutions. In Section 4 we depict the structure of the flow induced by the impact. We deduce that the slip velocity is logarithmically divergent close to the contact line. The momentum's lost due to the impact is also computed. Finally, in Section 5 we summarize our main results. The solution of several two-dimensional impact problems for planar and circular geometries is presented in appendix. Each problem is solved with a different method.

2. Model

2.1. Pressure impulse theory

Since the pioneering works of Wagner [2] on the landing of seaplanes, impact problems were enlightened by the concept of pressure impulse introduced by Bagnold [16]. Otherwise this quantity was used by Lamb [17] in order to develop a mathematical model of suddenly changed flow. The idea is to notice that a sudden change of the motion of one of the boundary of the fluid domain induces pressure gradients which in turn produce a sudden change in the liquid velocity [18]. This change occurs over a timescale τ very small compared to the convective time R/U. Therefore, by introducing a small parameter $\varepsilon = \frac{\tau}{R/U}$, we deduce after a comparison of the order of magnitude of each terms of the momentum equation that the non-linear terms are negligible compared to the time derivative of the velocity. In this study we only consider inertia-dominated impact then we assume that gravity, capillary and viscous effects are small with respect to inertial ones, *i.e.* Froude $Fr = U^2/gR$, Weber $We = \rho U^2 R/\sigma$ and Reynolds $Re = \rho U R/\mu$ numbers are all large with respect to unity. Here g denotes the gravity, σ the liquid-gas surface tension, ρ the liquid density and μ its viscosity. Consequently the time derivative of the velocity is just balanced by the pressure gradient. Then, at leading-order, the problem is described by the following equation [17,18] :

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla p, \tag{1}$$

where p is the pressure of the liquid. We assume here that the atmospheric pressure is the reference pressure. By integration of the relation (1) on the impact duration, we obtain:

$$\boldsymbol{u}_{impact} = \boldsymbol{u}(\tau) - \boldsymbol{u}(0) = -\frac{1}{\rho}\nabla P, \tag{2}$$

with P the pressure impulse defined by:

$$P = \int_0^\tau p(\boldsymbol{x}, t)\mathrm{d}t.$$

The impact velocity considered in this paper is assumed much lower than the speed of sound c. Hence we can suppose that the flow induced by the impact is incompressible. Therefore by taking the divergence of (2) we deduce that the pressure impulse satisfies Laplace's equation $\Delta P = 0$.

The problem as described here is general and at this stage the pressure impulse theory could be applied to many problems whatever the geometry *e.g.* with a complete sphere for drop deformation by laser-pulse impact [19] or with a plane for the impact of a wave on a seawall [1]. However the mathematical form of the solution strongly depends on the geometry and on the boundary conditions.

2.2. Problem statement

We consider a perfectly hemispherical drop of density ρ, surface tension σ and radius R, lower than the gravity–capillary length $l_{gc} = \sqrt{\sigma/\rho g}$, disposed on a circular cylinder. The base of the hemisphere coincides with the circular disc at the top of the cylinder whose radius is at least R. The cylinder impulsively starts from rest with a velocity U toward the drop or equivalently the cylinder falls in free fall and imposes a velocity U to the drop when that one hits the soil. Hence the impact induces a flow assumed axisymmetric and inviscid. As shown in the previous paragraph the impulsive problem is reduced to the derivation of Laplace's equation, in spherical coordinates in the present case:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial P}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right) = 0. \tag{3}$$

This equation is completed by *(i)* a dynamical condition which represents normal stress continuity at the free surface $\mathcal{S}$ and which takes account of the high Weber number hypothesis and *(ii)* a condition expressing the sudden variation of velocity $(\boldsymbol{u}\cdot\boldsymbol{e}_z)|_{z=0} = U$ at the bottom of the drop $\mathcal{P}$ at the impact moment, where $\boldsymbol{e}_z$

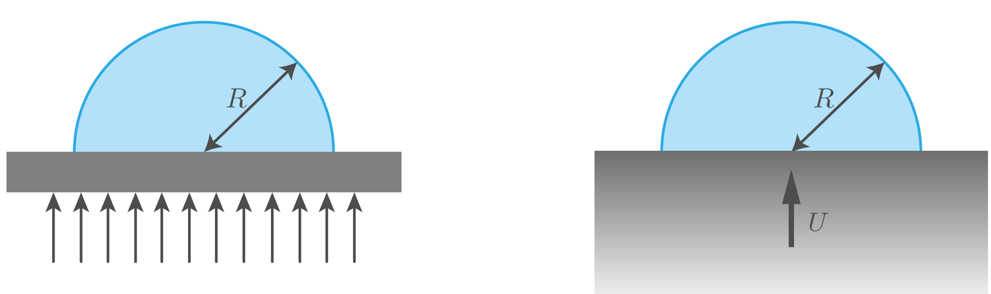

Fig. 2. Left: Sketch of a drop of radius R sitting on a solid substrate. The latter is initially impacted from beneath. The impulsion induces deformations of the drop and eventually its partial or total ejection if the impact energy is sufficiently large. Right: Sketch of the problem studied in this paper. A drop is disposed on an nondeformable cylindrical substrate of larger radius. The latter rises impulsively with a velocity U toward the drop at time $t = 0$. This problem could also be seen as an impact problem by considering the free fall of a cylinder with a drop disposed at its top. The substrate rises at a velocity U in the reference frame of the drop as soon as the cylinder hits the soil.