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Nonlinear evolution equations in crossing seas in the presence of uniform wind flow

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The effect of uniform wind flow in a situation of crossing sea states is studied here on the stability analysis for a pair of obliquely interacting uniform wave trains. It is observed that the growth rate of instability in crossing seas in the presence of wind flow is much higher than that in the absence of wind flow. It is also found that the growth rate of instability increases as the wind flow velocity increases up to some critical velocity beyond which the wave becomes linearly unstable and consequently the present analysis does not remain valid. The growth rate of instability obtained here is also greater than that for a single wave packet propagating in the presence of uniform wind flow.

1. Introduction

The problem of evolution of one surface wave packet in the presence of another surface wave packet has been considered by many authors [1–10] in different contexts. The two wave packets may be co-propagating waves or counter-propagating waves or obliquely propagating waves. Onorato et al. [1] studied nonlinear interaction of two co-propagating waves in shallow water. They showed that a system of defocusing coupled nonlinear Schrödinger equations can exhibit modulational instability if higher order dispersive terms are retained in the system. Debsarma and Das [2] also considered two co-propagating capillary–gravity wave packets. They derived fourth order non-linear evolution equations for the two wave packets and showed that the growth rate of instability of a uniform surface gravity wave train increases due to the presence of a capillary–gravity wave train. Nonlinear evolution of counter-propagating wave packets was considered by Pierce and Knobloch [3], Debsarma and Das [4]. Pierce and Knobloch [3] derived third order nonlocal mean-field evolution equations for two counter-propagating capillary–gravity wave packets on the surface of water of finite depth. Debsarma and Das [4] considered modification of this water wave model by including fourth order nonlinear terms in the evolution equations for the case of infinite depth water and carried out stability analysis of a uniform standing wave train under longitudinal perturbations. Dhar and Das [5] carried out stability analysis of a uniform Stokes wave train in the presence of another obliquely propagating Stokes wave train assuming that the space variation of amplitude takes place only in a direction along which the group velocity projection of the two waves overlap. They observed significant deviations in the growth rate of instability at fourth order compared to that at the third order.

In recent years, the dynamics of two obliquely interacting wave systems have become an important topic of investigation. The reason is that the growth rate of modulational instability for a pair of obliquely interacting uniform wave trains is much larger than that for a single wave train. This was first shown by Onorato et al. [6]. They carried out stability analysis for unidirectional perturbations only. Later on, Shukla et al. [7] extended the analysis to incorporate bi-directional perturbations. Laine-Pearson [8] also studied weakly nonlinear interaction of two-wave systems propagating in different directions on deep water. He showed that the growth rate of long wave instability of two obliquely interacting waves can be larger than those for resonant interaction of short-crested waves. Because of higher growth rates in crossing sea states some of the authors [Onorato et al. [6], Laine-Pearson [8]] have suggested modulational instability as a possible mechanism for the formation of large amplitude rogue waves in crossing sea states. The evolution

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equations derived by Onorato et al. [6], Shukla et al. [7], Laine-Pearson [8] are at the lowest order, i.e., at the third order of nonlinearity and under constant atmospheric pressure. The effect of higher order nonlinearity on modulational instability in crossing sea states is studied in Gramstad and Trulsen [9] and also in Debsarma et al. [10].

Ruban [11] performed numerical experiments for long-crested water waves assuming quasi-random initial conditions. As an initial state, he considered a superposition of quasi-randomly placed wave packets which include 25 wave packets having wave number vector (60,2), 25 packets having wave vector (50,0), 16 packets with (40,−2), and 12 packets with (30,1). In his experiment, a single rogue wave was formed spontaneously which existed for several wave periods without significant change of its amplitude. In another experiment, Ruban [12] considered crossing sea states situation defined by two wave vectors $\overrightarrow{k}_0 - \nabla\overrightarrow{k}/2$ and $\overrightarrow{k}_0 + \nabla\overrightarrow{k}/2$ where $|\nabla\overrightarrow{k}| \ll |\overrightarrow{k}_0|$. He observed two different kinds of rogue waves forming in two different situations depending on the angle between the vectors $\overrightarrow{k}_0$ and $\nabla\overrightarrow{k}$. Some of the recent developments of rogue wave phenomenon have been reported by 18 authors including Ruban et al. [13] in the special issue of the European Physical Journal, July 2010.

In the present paper, we have considered crossing sea states situation in the presence of uniform wind blowing over water of infinite depth. Two gravity wave packets are propagating obliquely at the air–water interface making equal angles with the direction of wind flow.

The mechanisms for the generation of ocean waves by wind and the coupling between ocean waves and wind has a long history. Miles [14] gave an explanation of the generation mechanism of water waves by considering a model of an inviscid shear flow in air over water. Extension of Miles theory and further developments on this topic have been presented very nicely in Phillips [15]. The effect of uniform wind flow on the propagation of a quasi-chromatic surface gravity wave packet has been studied by Dhar and Das [16]. The model presented here is an extension of their work to the situation of crossing sea states. Brunetti et al. [17] derived wind forced nonlinear Schrödinger equation under the assumption that the growth rate of the waves is of the order of wave steepness. Asymptotic stability of wind forced modulations in a situation of crossing sea states is recently investigated in Debsarma et al. [18,19].

The paper is organized as follows: In Section 2, we have written down the basic equations governing the model as mentioned above. In Section 3, we have derived evolution equations of the two wave packets. The evolution equations derived here remain valid as long as the wind velocity is less than a critical velocity. The reason is that a wave becomes linearly unstable when the wind velocity is greater than this critical velocity. For the linearly stable case, two different modes of wave propagation are possible. We call them as positive and negative modes. In Section 4, we have made stability analysis of two wind-flow modified uniform wave trains. We have plotted growth rate of instability in the perturbed wave number plane for different values of θ, twice of which is the angle between the directions of propagation of the two wave packets. The growth rate of instability have been shown in figures for both of positive and negative modes of wave propagation.

2. Governing equations

We consider crossing seas in the presence of wind blowing uniformly over water. We assume that before the onset of water wave motion wind flows over water with a constant velocity U in a direction parallel to x-axis. The undisturbed interface between air and water is taken as the xy-plane and z-axis is taken positive vertically upwards. We take $z = \zeta(x, y, t)$ to be the equation to the wavy interface at any time t. Let ρ_w and ρ_a be the densities of water and air, respectively. The perturbed velocity potentials ϕ_w and ϕ_a of water and air, respectively, satisfy Laplace equations.

$$\nabla^2\phi_w = 0, \quad -\infty < z < \zeta \tag{1a}$$

$$\nabla^2\phi_a = 0, \quad \zeta < z < \infty. \tag{1b}$$

The kinematic boundary condition at the interface is

$$\frac{\partial\phi_w}{\partial z} - \frac{\partial\zeta}{\partial t} = \frac{\partial\phi_w}{\partial x}\cdot\frac{\partial\zeta}{\partial x} + \frac{\partial\phi_w}{\partial y}\cdot\frac{\partial\zeta}{\partial y} \quad \text{on } z = \zeta \tag{2a}$$

$$\frac{\partial\phi_a}{\partial z} - \frac{\partial\zeta}{\partial t} - U\frac{\partial\zeta}{\partial x} = \frac{\partial\phi_a}{\partial x}\cdot\frac{\partial\zeta}{\partial x} + \frac{\partial\phi_a}{\partial y}\cdot\frac{\partial\zeta}{\partial y} \quad \text{on } z = \zeta. \tag{2b}$$

The dynamic boundary condition to be satisfied at the interface is

$$\frac{\partial\phi_w}{\partial t} - r\frac{\partial\phi_a}{\partial t} + g(1-r)\zeta - rU\frac{\partial\phi_a}{\partial x} = -\frac{1}{2}\left(\vec{\nabla}\phi_w\right)^2 + \frac{1}{2}r\left(\vec{\nabla}\phi_a\right)^2 \quad \text{on } z = \zeta \tag{3}$$

where $r = \rho_a/\rho_w$. At infinity, the velocity potentials ϕ_w and ϕ_a satisfy the following conditions:

$$\phi_w \to 0 \quad \text{as } z \to -\infty \tag{4a}$$

$$\phi_a \to 0 \quad \text{as } z \to \infty. \tag{4b}$$

Considering the linearized form of Eqs. (1)–(4), we see that the wave number (k, l) and frequency ω of a wave propagating at the air–water interface satisfy the following linear dispersion relation:

$$D(\omega, k, l) \equiv \omega^2 + r(\omega - Uk)^2 - g(1-r)\sqrt{k^2 + l^2} = 0. \tag{5}$$

Eq. (5) gives the following two values of ω:

$$\omega_\pm = \frac{rUk \pm \sqrt{(1-r^2)gk_0 - rU^2k^2}}{1+r} \tag{6}$$

where $k_0 = \sqrt{(k^2 + l^2)}$. If the wave propagates with frequency $\omega = \omega_+$ (or ω_-), we call it to be positive mode (or negative mode) of wave propagation. Eq. (6) also shows that a wave of wave-number (k, l) becomes linearly unstable if the wind flow velocity U satisfies the following condition:

$$|U| > U_c \quad \text{where } U_c = \sqrt{\frac{(1-r^2)gk_0}{rk^2}}. \tag{7a}$$

It is important to note that the critical velocity depends on the carrier wave number k_0 of either wave packet. Setting $k = k_0\cos\theta$ and $l = k_0\sin\theta$, we find from (7a) that

$$\frac{1}{\sqrt{k_0}} = U_c\cos\theta\sqrt{\frac{r}{(1-r^2)g}} \tag{7b}$$

where θ is the angle made by the direction of propagation of either wave packet with the direction of wind velocity. Since, for a fixed θ, U_c is inversely proportional to the square root of k_0, U_c is quite large when k_0 is small. Thus, there exists a wide range of wind velocity U for which there is no linear instability of waves. In Fig. 1, we have plotted dimensional values of U_c against k_0 for different values of θ, using Eq. (7b). It is reported in Kjeldsen [20] that during the accident of Norwegian ship 'NORSE VARIANT' in 1973 there was a strong northerly wind with wind velocity near to 60 knots[1 knot = 0.514 m/s]. In Proudman [21], some observed values of wind speed over North Atlantic, North Pacific, South Pacific, and Indian Ocean are given which lie in the range 7.8–23 cm/s. In our model, we assume that the wind flows with a velocity U whose magnitude is much less than the critical value U_c. Under such an assumption, we have investigated modulational instability of a pair of uniform wave trains in Section 4.