European Journal of Mechanics B/Fluids 60 (2016) 119–126

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

A homotopy series solution to a nonlinear partial differential equation arising from a mathematical model of the counter-current imbibition phenomenon in a heterogeneous porous medium

Kajal K. Patel *, M.N. Mehta, Twinkle R. Singh

Department of Applied Mathematics & Humanities, S. V. National Institute of Technology, Surat-395007, India

ARTICLE INFO

Article history:
Received 17 February 2014
Received in revised form
30 May 2016
Accepted 11 July 2016
Available online 26 July 2016

Keywords:
Imbibition
Heterogeneous porous media
Homotopy analysis method

ABSTRACT

The mathematical model describing the counter-current imbibition phenomenon in a heterogeneous porous medium gives rise to a non-linear partial differential equation. This equation has been solved using homotopy analysis methods together with appropriate boundary and initial conditions. The solution represents the saturation of injected water during counter-current imbibition in a secondary oil recovery process, when water is injected. This saturation of injected water increases as the distance, X, from the imbibition phase increases, for a given time $T > 0$. It is also deduced that the saturation of injected water in a homogeneous porous matrix is higher than the saturation of injected water in a heterogeneous porous matrix for the same distance X and time $T > 0$. The numerical and graphical presentation of the saturation of injected water in heterogeneous as well as homogeneous porous matrices for distance X and time $T > 0$ are obtained in Maple.

1. Introduction

The study of injected water in heterogeneous, as well as fractured porous, media offers special challenges to mathematicians and engineers. It is relevant to the secondary oil recovery process, in which counter-current imbibition occurs at the common interface of an oil formatted area and a water injection area [1,2]. In a heterogeneous porous matrix, the physical properties of the material are significantly affected by the flow of fluid during the secondary oil recovery process. Our particular interest is to study the counter-current imbibition phenomenon, which occurs during the secondary oil recovery process, for oil displacement due to the water injection. The investigation of oil recovery via the spontaneous imbibition phenomenon has been a highly influential area of study within the field of multiphase flow in porous media [3]. This study can be carried out by selecting a small model from a large natural oil field. The classical model of the simultaneous flow of immiscible fluids in porous media was constructed in the late 30s and early 40s by distinguished American scientists and engineers. The Muskat–Leverett theory [4,3] was of fundamental importance to the development of oil recovery processes. Oil recovery from fractured reservoirs by means of the imbibition mechanism is also significant to research in porous media water flooding processes. Fractured reservoirs are composed of fracture systems and a matrix. The fractures have higher permeability and relatively low volume compared to the matrix, the permeability of which is very low, as it contains the majority of the oil. Water flooding is used to increase oil recovery by increasing water pressure in fractures since water quickly surrounds the oil-saturated matrix of lower permeability. The process of water flooding works well when the matrix is water wet and imbibition can lead to significant recoveries, while poor recoveries and early water breakthrough occur when oil-wet matrix conditions prevail. Imbibition is defined as the displacement of the non-wetting phase (oil) by the wetting-phase (water) where the dominant effect is the capillary force. Imbibition can occur in both counter-current and co-current flow models depending on the fracture network as well as the heterogeneous porous matrix and water injection rates. In co-current imbibition, water displaces oil out of the matrix thus both water and oil flow in the same direction. Counter-current imbibition, on the other hand, is where the wetting phase imbibes. The latter is open only for displacement mechanisms where a region of the heterogeneous porous matrix is exposed on one side to a water filling area [5–7].

Imbibition has also been investigated by several other authors, either co-current, counter-current, or both of them together [8–10]. Reis and Cil [11] introduced a one-dimensional model for

* Correspondence to: 20A Mayfield Street, Westminster, WA 6061, Australia.
E-mail address: kajal11@gmail.com (K.K. Patel).

http://dx.doi.org/10.1016/j.euromechflu.2016.07.005
0997-7546/

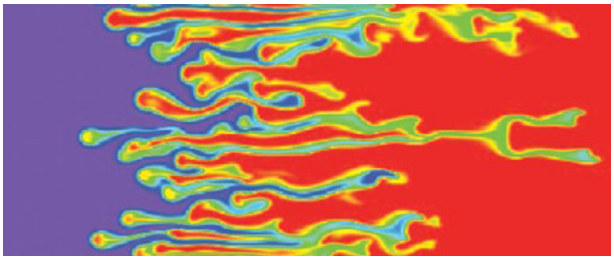

Fig. 1. Representation of counter-current imbibition phenomenon, when non-wetting oil and wetting water contact in each other without any external force in a heterogeneous porous matrix.

expulsion by countercurrent water imbibition in rock. Marrow and Mason [12] presented a comprehensive review on the recovery of oil by spontaneous imbibition. Kashchiev and Firoozabadi [10] gave analytical solutions for one-dimensional counter-current imbibition in a water-wet matrix. The analytical study of oil recovery during counter-current imbibition in strongly water-wet systems was given by Tavassoli et al. [13]. In most of the previously mentioned imbibition studies, the researchers have neglected the effect of gravity by dropping the gravitational force term from the flow equations, especially for the oil–water modeling. Analytical and numerical solutions of gravity–imbibition and gravity-drainage processes were given by Bech et al. [5]. Ruth et al. [14] provided an approximate analytical solution for counter-current spontaneous imbibition. In spontaneous imbibition, capillary suction causes the wetting fluid to be drawn into the rock, and the non-wetting fluid moves out as shown in Fig. 1.

When water flooding or water injection occurs in a large oil formatted natural area, then oil is displaced by water injection at the common interface where water comes into contact with the oil formatted area without any external force instead of regular displacement of the whole front protuberance occurs. The occurrence of protuberances looks like shape of multi branch channels or "fingers" as shown in Fig. 2. The microscopic behaviour of these fingers is analysed by the statistical treatment [15] at common interface $x = 0$.

This phenomenon was analytically discussed for the first time by Scheidegger and Johnson [15]. They have described a different approach to this phenomenon which they have called the statistics of fingers as shown in Fig. 3. Verma [16] has obtained a solution for the imbibition phenomenon in cracked porous media. Patel and Mehta [2] have studied this phenomenon by converting the governing equation to the form of Burger's equation. Kinjal and Mehta [7] have obtained a series solution for the imbibition phenomenon in heterogeneous porous media arising in the secondary oil recovery process. Meher and Mehta have solved this phenomenon via the Adomian decomposition method, obtaining an analytical series solution of double phase flow through homogeneous porous media with capillary pressure [17]. Liao [18] applied the basic idea of homotopy, from topology, to propose a general analytical method for nonlinear problems. This method has been successfully applied to solve many types of linear and nonlinear differential equations in various fields of engineering and science [19,18,17]. Cheng J. et al. [20] have used HAM to find the solution of an explicit series approximation to the optimal exercise of moving boundary problems by means of a new analytical method for strongly nonlinear problems. This approach is general and can be applied to many other moving boundary problems in finance and engineering. Liao [4] found that all optimal homotopy-analysis approaches greatly accelerate the convergence of series solutions and can be used to get quickly converging series solutions to different types of equations with strong nonlinearity. This differs from the approach that focuses on the non-linearity of the equations. The homotopy analysis method is a powerful technique to solve nonlinear differential equations in fluid dynamics problems [19,17]. The key to this method is the generalized Taylor expansion; Liao points out that generalized Taylor series provide a way to control and adjust the convergence region through an auxiliary parameter $\hbar$ such that the Homotopy analysis method is particularly suitable for problems with strong nonlinearity [21,18].

Thus the simultaneous flow of two immiscible fluids in a heterogeneous porous matrix has great importance in an oil saturated reservoir. It can be simply defined as when any two immiscible fluids come into contact naturally or artificially with or without any external forces. Then there are two types of imbibition, as discussed earlier: co-current and counter-current imbibition. It is known that in co-current imbibition the flow of the wetting fluid (water) and the non-wetting fluid (oil) are in the same direction. For counter-current imbibition, the wetting and non-wetting fluids flow in opposite direction, so $V_{iw} = -V_{no}$ [15] for a short distance and for a small time, which is called the imbibition condition. In the present paper, we have studied counter-current imbibition in a heterogeneous porous matrix by choosing a mathematical model. In counter-current flow, the gravitational and capillary forces have a less significant role. A new approach to the problem is proposed in Barenblatt et al. (2003), who suggest a theory of multiphase flow with a relaxation time to explain counter-current imbibition. The Barenblatt model of spontaneous counter-current imbibition was investigated by Silin and Patzek [22]. But in the present analysis such effects are ignored, and the classical approach has been taken, and therefore it is necessary to assume that a Darcy-like formulation of the flow equation is sufficient for an analysis in which the local wetting fluid saturation $S_{iw}(x, t)$ obeys a differential equation of the diffusion type and the capillary diffusion coefficient $D(S_{iw})$, is directly related to the relative permeability and capillary pressure functions. In the present work spontaneous imbibition has been studied in a heterogeneous porous matrix and we have also obtained a result for a homogeneous porous matrix. In a heterogeneous porous matrix, porosity and permeability may vary from one place to another and also other factors like fluid interfacial tension, the initial water saturation and relative permeability affect the flow rates and the displacement of the oil towards the production wells. It is a counter-current imbibition process as wetting and non-wetting fluids flow in opposite directions, thus the sum of the Darcy velocities is taken to be zero. Also, specific results for the dependence of relative permeability and capillary pressure on phase saturation have been assumed from standard literature. Governing nonlinear partial differential equations have been formulated and solved by using the homotopy analysis method.

1.1. Statement of the problem

When water flooding or water injection occurs in a large oil formatted natural area, then oil is displaced by water injection at the common interface where water comes into contact with the oil formatted a counter-current imbibition occurs at the common interface, which satisfies the imbibition condition $V_{iw} = -V_{no}$ [15]. The mathematical model of this unsteady flow is based on the fundamental assumption of local phase equilibrium. In this model Darcy's law [23] and the equation of continuity for water and oil have been used. As governing equations in the formulation of the model, some standard relationships between capillary pressure and saturation are also considered in the model. This two-phase flow model is based on the following assumptions.

- Pore air and pour water are single component fluids.
- Mass transfers between the fluids, i.e. the dissolution of air in water and the evaporation of water, are neglected.