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The influence of an interfacial heat release on nonlinear traveling waves in a two-layer system under the action of an imposed temperature gradient

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HIGHLIGHTS

- The influence of an interfacial heat release on nonlinear traveling waves has been studied.
- The joint action of buoyancy and thermocapillary effect has been investigated.
- The period-three phase trajectory has been found.
- New oscillatory regimes have been obtained.

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ABSTRACT

The influence of an interfacial heat release on nonlinear traveling waves, developed under the action of an imposed temperature gradient in the 47v2 silicone oil–water system, has been studied. Transitions between the flows with different spatial structures, have been investigated. It is shown that the presence of an interfacial heat release can change the sequence of bifurcations and lead to the appearance of new oscillatory regimes. The period-three phase trajectory has been found.

1. Introduction

Interfacial convection in systems with interfaces has been a subject of an extensive investigation at the past few decades (for a review, see [1,2]). Traditional fields of application of the interfacial convection are chemical engineering [3] and materials processing [4]. Among the modern techniques requiring an investigation of convection in systems with interfaces one can mention liquid encapsulation crystal growth technique used in space laboratories missions [5], droplet–droplet coalescence processes, where the Marangoni convection in the inter-droplet film can considerably affect the coalescence time during extraction [6], and others.

It is known that the stability problem for the mechanical equilibrium in a system with an interface is not self-adjoint (see [1,7]), thus an oscillatory instability is possible. The mechanism of oscillations, which develops without interfacial deformations due to the hydrodynamic and thermal interaction between convective flows on both sides of the interface, was found by Gershuni and Zhukhovitsky [8] in the case of transformer oil–formic acid system. However, the threshold value of the Rayleigh number for the oscillatory instability was higher than that for the monotonic instability. According to Gilev et al. (see [9]), if the ratio of the dynamic viscosities of fluids is reduced, the neutral curve has its minimum for oscillatory disturbances (see also [1]). Renardy [7] found that the same result can be obtained in the silicone oil–Fluorinert system if instead the "true" Fluorinert, a *fictitious* fluid with an enhanced thermal diffusivity of Fluorinert, is considered. The nonlinear oscillatory convective structures near the instability threshold for some model systems have been studied in [10,11].

An oscillatory instability of the mechanical equilibrium can be caused by the joint action of buoyancy and thermocapillary effect in a two-layer system heated from below. This phenomenon

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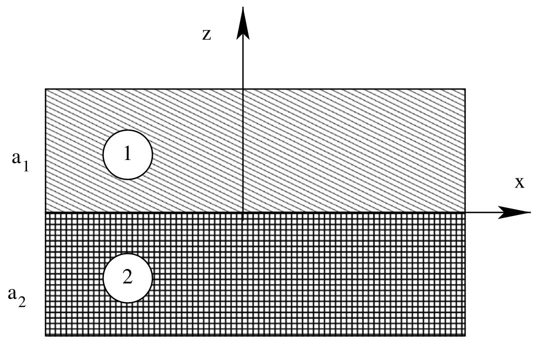

Fig. 1. Geometrical configuration of the two-layer system and coordinate axes.

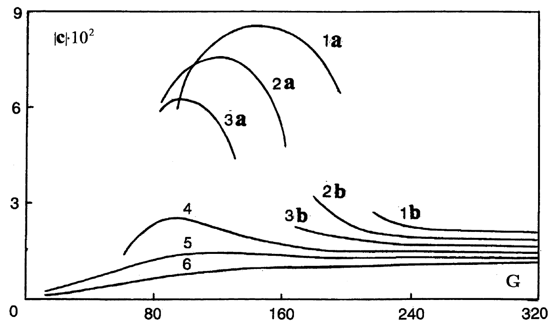

Fig. 2. The dependences of the phase velocity $|c|$ on the Grashof number G at $G_Q = 0$ (lines 1a, 1b); -25 (lines 2a, 2b); -50 (lines 3a, 3b); -100 (line 4); -150 (line 5); -300 (line 6); $K = 0.025$; $L = 2.74$; $\epsilon = 0.01$; $a = 1$.

was described in [12] (see also [1,13]). Oscillations just above the instability threshold have been observed in experiments [14].

Under experimental conditions, the temperature gradient is not perfectly vertical and the horizontal component of the temperature gradient appears. The appearance of this component changes the situation completely: at any small values of the Marangoni number ($M \neq 0$), the mechanical equilibrium becomes impossible, and a convective flow takes place in the system. Thus, it is reasonable to consider the influence of the horizontal component of the temperature gradient on convective regimes developed in the system (see [15,16]). The Marangoni convection under the action of the inclined temperature gradient in two-layer systems, where the instability of a thermocapillary flow manifests itself in the form of hydrothermal waves or convective patterns, has been investigated in [17]. The influence of the horizontal component of the temperature gradient on convective oscillations in a two-layer system filling a closed cavity with *rigid heat-insulated lateral walls*, has been considered in [18]. It was shown that the horizontal component of the temperature gradient could lead to the violation of the symmetry conditions and the appearance of asymmetric oscillatory flows.

The interaction between the convection caused by the temperature gradient directed perpendicularly to the interface and the convective flow produced by the horizontal component of the temperature gradient in *a laterally infinite two-layer system* in the presence of buoyant and thermocapillary effects, has been studied in [19]. Specifically, two types of traveling waves—a "fast" traveling wave, moving in the direction of an imposed temperature gradient and a "slow" wave, moving in the opposite direction, have been obtained. A pulsating traveling wave changing the direction of propagation during one period, has been observed in [19]. This wave has been found in the interval of the Grashof number values, bounded from below and from above by the regions of "fast" and "slow" traveling waves.

There are various physical phenomena that can be the origin of a heat release on the interface. The interfacial heating can be generated, e.g., by an infrared light source. The infrared absorption bands of water and silicone fluids are essentially different [20], therefore the light frequency can be chosen in a way that one of the fluids is transparent, while the characteristic length of the light absorption in another liquid is short. The presence of a constant, spatially uniform heat release at the interface can lead to the appearance of an oscillatory instability [21]. The influence of an interfacial heat release on convective oscillations in the case of *rigid heat-insulated lateral walls* corresponding to a closed cavity, has been studied in [22,23]. In [22] the external temperature gradient was directed perpendicularly to the interface and in [23] convective flows have been produced by the horizontal component of the temperature gradient.

Let us note that the theoretical predictions obtained for flows in closed cavities in [22,23] cannot be automatically applied for the infinite layers because of several reasons. First, for the observation of waves in a closed cavity a global instability is needed, while in the case of periodic boundary conditions one observes waves generated by a convective instability of a parallel flow [24]. Also, in the presence of rigid lateral walls the basic flow is not parallel—the lateral walls act as a stationary finite-amplitude perturbation that can produce steady multicellular flow in the part of the cavity and in the whole cavity.

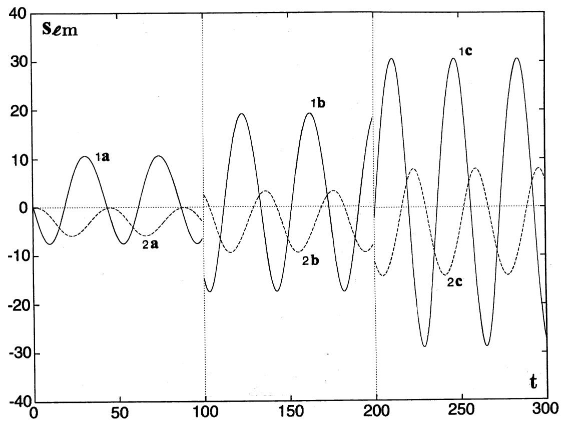

Fig. 3. Dependences of $S_{l,m}$ on time ($m = 1, 2$) for the traveling wave moving from the right to the left at $G = 87.2$ (lines 1a, 2a); 95.2 (lines 1b, 2b); 112 (lines 1c, 2c); $K = 0.025$; $G_Q = -25$; $L = 2.74$; $\epsilon = 0.01$; $a = 1$.

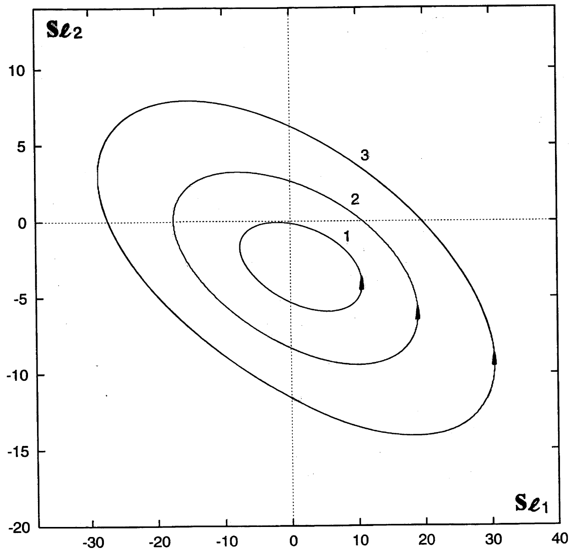

Fig. 4. Phase trajectories in the plane (S_{l1}, S_{l2}) at $G = 87.2$ (line 1); 95.2 (line 2); 112.0 (line 3); $K = 0.025$; $G_Q = -25$; $L = 2.74$; $\epsilon = 0.01$; $a = 1$.