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Theoretical study of the energy harvesting of a cantilever with attached prism under aeroelastic galloping

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ABSTRACT

The aeroelastic galloping of a cantilever with attached prism has recently attracted the attention of several researchers as a way to harvest energy from an airstream. This arrangement is not entirely analogous to that of classical Transverse Galloping (TG) since the instantaneous attitude of the galloping body (prism) with respect to the incident flow depends both on the velocity of the galloping body and wind speed (like in TG) but also on the rotation angle at the cantilever free end. A new governing parameter emerges, namely the ratio of the cross-section length of the prism to the beam length δ, and its effect on the galloping dynamics and power output needs to be studied. To this end, a theoretical model is here developed where the influence of δ is considered.

1. Introduction

Flow-induced oscillations by Transverse Galloping (TG) were pointed out by Barrero-Gil et al. [1] as a potential source for energy harvesting from an airstream. TG is a fluid-elastic instability that appears in some elastic bluff bodies when the velocity of the incident flow exceeds a critical value. Then, oscillatory motion (transverse to the flow) develops with increasing amplitude until the energy dissipated per cycle by mechanical damping balances the energy input per cycle from the flow (for a detailed introduction to TG the reader is referred to Parkinson [2], or Paidoussis et al. [3]). If the geometry of the body and the elastic properties are appropriate, the TG instability may appear at low flow velocities and with large excitation amplitudes, making TG a very promising way to harvest energy successfully [1,4,5].

Barrero-Gil et al. [1] made an analytical treatment to give the level of mechanical power extraction as a function of the geometry of the cross-section of the galloping body, its mechanical parameters, and flow velocity. Findings like the maximum efficiency achievable or the wind speed at which this maximum occurs were reported. Since then, several researches have studied how to implement the concept in a real energy harvester, with emphasis in low power generation systems, of the order of milli-Watts or tens of milli-Watts (see, for example, Sirohi and Mahadik [6,7], Zhao et al. [8], Yang et al. [9], Xu-Xu et al. [10]), with characteristic dimensions of the order of centimeters. They have been focused on experimental arrangements where a rigid galloping body is fixed to the free end of a cantilevered beam (see Fig. 1). For electricity conversion piezoelectric sheets are usually attached to the base of the beam. Under the effect of an airstream, for high enough air speed, oscillations by galloping take place and the induced strain in the piezoelectric patches produces an electrical current which is dissipated at the electrical load R_L (see, for example, Yang et al. [9]). However, in this cantilevered arrangement the situation is not entirely analogous to that of pure TG analyzed in Barrero-Gil et al. [1], since the instantaneous attitude of the galloping body with respect to the incident flow depends on the velocity of galloping body and wind speed (like in TG) but also on the rotation angle at the beam free end (see Fig. 1; Kluger et al. [11]). A new governing parameter appears, namely the ratio of the cross-section characteristic length D to the cantilever beam length L_b, defined as $\delta = 3D/(2L_b)$, and its role on the dynamics of the body and electrical power output should be studied. With this idea in mind, we present here a theoretical model of a generic energy harvester where the galloping body is cantilevered mounted. Quasi-steady conditions are assumed to model aerodynamic forces and a kinematic relationship is introduced for the instantaneous angle of attack where rotation of the beam is considered. An equivalent circuit model is employed for the piezoelectric sheets. The mathematical model is approximately solved by applying the standard Harmonic Balance Method, and analyzed in detail. The analysis is focused on the

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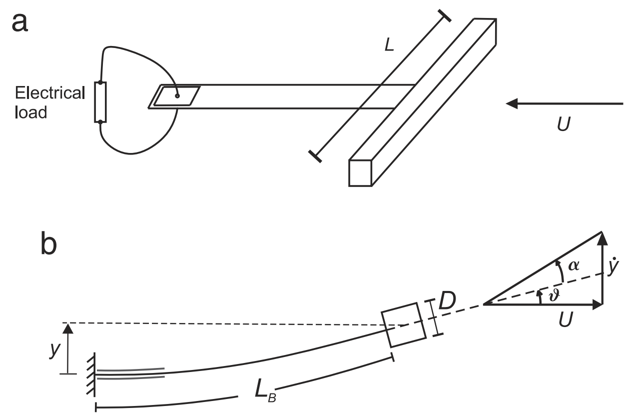

Fig. 1. (a) Typical arrangement piezoelectric energy harvesting from galloping of a cantilevered prism. (b) One-degree-of-freedom model: vertical displacement of the rigid bluff body.

impact of the system dimensionless parameters on the efficiency of energy harvested. As a novelty, the analysis allows to see clearly that δ has a negative impact into the electrical power output.

First of all, in Section 2, an electro-aero-elastic model is introduced. An analytical approximate solution is found in Section 3 that allows us to get physical insight and to discuss the influence of the length of the beam and other governing parameters on both the galloping body dynamics and electrical power. Analytical predictions are compared with experimental results from Zhao et al. [8] in Section 4. Good agreement is found. Finally, concluding remarks are drawn in Section 5.

2. Theoretical model

Let us introduce a one-degree-of-freedom model to describe the transverse displacement of the prism shown in Fig. 1. It is based on the equilibrium between inertia, damping, and stiffness forces, as well as vertical aerodynamic force, and the electromechanical force induced by the piezoelectric transducer. That is,

$$m\ddot{y} + c\dot{y} + ky = \frac{1}{2}\rho U^2 DLC_Y - F_p, \tag{1}$$

where y denotes the transverse position of the prism, m is the equivalent mass of the prism, c is an equivalent damping constant, k is the equivalent stiffness constant, ρ is the fluid density, U is the undisturbed velocity of the incident flow, D the side length of the prism's cross-section and L its length, C_Y is the instantaneous aerodynamic force coefficient in the transverse direction to the incident flow, and F_p is the electromechanical force in the y direction due to the piezoelectric effect. Finally, the dot symbol stands for differentiation with respect to physical time t.

The equivalent (or effective) mass of the prism is given by the prism mass plus the effective mass of cantilever beam. The effective mass of cantilever beam can be approximated as 0.25 times the mass of the cantilever beam [12]. The equivalent damping and stiffness constants can be obtained experimentally from a free decay tests in absence of fluid flow by measuring the decay rate of the amplitude and frequency of oscillations.

Note that, for the sake of simplicity, damping and stiffness forces have been considered linear, which is a realistic approximation when transverse displacements of the prism are small compared to the length of the beam.

2.1. Aerodynamic force

In order to describe C_Y, the quasi-steady hypothesis is usually resorted to (see Paidoussis et al. [2]), since galloping is typically a low-frequency oscillation phenomenon where the characteristic timescale of the prism oscillation (of order $2\pi(m/k)^{1/2}$) is much larger than the characteristic timescale of the flow (of order D/U). Then, the aerodynamic force is only dependent on the instantaneous attitude of the prism with respect to the incident flow, which can be described by the effective angle of attack α. From Fig. 1(b),

$$\tan(\alpha + \theta) = \frac{\tan\alpha + \tan\theta}{1 - \tan\alpha\tan\theta}, \tag{2}$$

where θ is the rotation angle at the free end of the beam; α and θ are positive in the counterclockwise direction. Assuming that α and θ are small it follows that

$$\tan(\alpha + \theta) \simeq \tan\alpha + \tan\theta. \tag{3}$$

For a uniform cantilevered beam, $\theta = 3y/(2L_b)$ (see Kluger et al. [11]), where L_b is the length of the beam. In addition, $\tan(\alpha+\theta) = \dot{y}/U$ so it follows that

$$\tan\alpha \simeq \frac{\dot{y}}{U} - \frac{3y}{2L_b}. \tag{4}$$

For our theoretical analysis, to maintain a compromise between development complexity and accuracy, a cubic polynomial can be considered enough (see Blevins [13, p. 130]) to approximate the vertical aerodynamic force coefficient dependence with $\tan\alpha$, so that

$$C_Y = a_1\tan\alpha + a_3(\tan\alpha)^3, \tag{5}$$

where a_1 (>0) and a_3 (<0) are the empirical coefficients to fit by a polynomial the C_Y versus $\tan(\alpha)$ dependence measured in static tests (normally in wind tunnel). The values of a1 and a3 depend on the cross-section geometry of the prism. We refer the reader to Blevins [13], Bokaian and Geoola [14] or Barrero-Gil et al. [1] in order to obtain more information and typical values. Then, the aerodynamic force coefficient is

$$C_Y = a_1\left(\frac{\dot{y}}{U} - \frac{3y}{2L_b}\right) + a_3\left(\frac{\dot{y}}{U} - \frac{3y}{2L_b}\right)^3, \tag{6}$$

which can be simplified to

$$C_Y = a_1\left(\frac{\dot{y}}{U} - \frac{3y}{2L_b}\right) + a_3\left(\left(\frac{\dot{y}}{U}\right)^3 + \frac{27y^2\dot{y}}{4L_b^2U}\right), \tag{7}$$

if nonlinear stiffness terms are neglected, which makes sense since their effect in the overall response is expected to be small when the bluff body is under the action of light fluids (airstreams). Let us discuss this point in detail by comparing the nonlinear stiffness fluid force $\widetilde{F}_Y$

$$\widetilde{F}_Y = \frac{1}{2}\rho U^2 DL\left(\frac{27y^3}{8L_b^3} - \frac{9\dot{y}^2y}{2U^2L_b}\right), \tag{8}$$

and the stiffness force $F_s = ky$. That is

$$\frac{\widetilde{F}_Y}{F_s} = \frac{\rho U^2 DL}{2k}\left(\frac{27y^2}{8L_b^3} - \frac{9\dot{y}^2}{2U^2L_b}\right). \tag{9}$$

Taking $y \sim A$, where A is the steady-state amplitude of oscillations, $\dot{y} \sim A\omega_N$ where $\omega_N^2 = k/m$ is the natural frequency of oscillations, it follows that

$$\frac{\widetilde{F}_Y}{F_s} \sim \frac{a_3}{2m^*}\left(A^{*2}\delta^3U^{*2} - 3A^{*2}\delta\right), \tag{10}$$

where $m^* = m/(\rho D^2 L)$ is the mass ratio, $A^* = A/D$ is the normalized steady state amplitude of oscillations, $\delta = 3D/(2L_b)$, and $U^* = U/(\omega_N D)$ is the reduced velocity. Note that nonlinear stiffness fluid force terms are expected to be negligible for large m^*, which is a common situation when the fluid is light (airstreams for example).