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Investigation of similarity criterion for gas flow field based on physical properties

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HIGHLIGHTS

- Investigation of similarity criterion for gas flow field based on physical properties is carried out systematically and comprehensively. The criterion can make two gas flow fields strictly similar. It would be helpful for the experimental study of complex phenomenon which is influenced by all the parameters of flow field, such as thermo-convective and flow separation.
- Similarity relationships of aerodynamic pressure and heat flux on solid surface between two similar flow fields are presented.
- Numerical studies are done to validate the criterion with Computational Fluid Dynamics (CFD) method. Supersonic flow without gravity, subsonic flow without gravity and subsonic flow with gravity are respectively used in the simulation. The calculation results are in good agreement with the theoretical results.

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ABSTRACT

An investigation of a new similarity criterion for gas flow field is conducted theoretically and numerically. The criterion is derived theoretically based on physical properties. It solves the problem of conflict between the similarity requirements, and can make two flow fields completely similar. Two theoretical analysis methods, the formulation analysis and dimensionless analysis methods, are used for the derivation, and the conclusions are coincident. The criterion would be helpful for the experimental study of complex phenomena influenced by all parameters of the flow field, such as thermo-convection and flow separation. Based on kinetic theory, similarity relationships of aerodynamic pressure and heat flux on solid surface between two similar flow fields are presented. Simulations of nitrogen flow over a sphere and a third-scale model are performed to validate the criterion with the Computational Fluid Dynamics (CFD) method. Supersonic flow without gravity, subsonic flow without gravity and subsonic flow with gravity are used in the simulation. The calculation results are in good agreement with the theoretical results.

1. Introduction

Experimental research is one of fundamental approaches for studying gas flow mechanisms. However, in many cases, it is very difficult or even impossible to perform experiments with a real object due to the huge cost, long test period or limited equipment capacity. A scaling technique, by which the performance of a prototype could be predicted from scale models, is an effective approach to solve this problem. This has been accepted and widely used in many studies: for example, scale aircraft models are often used in wind tunnel experiments for the studies of lift on aircraft [1–10]; scale rocket engine models are used in experimental studies of the flow characteristics in combustion [11–16]; and scale building models are used to study the indoor air motions or urban air quality [17–25], etc.

The theoretical basis of the scaling technique is the similarity criterion. In the past few decades, much work has been carried out to study similarity relations, and many criteria have been proposed, such as the Reynolds number similarity criterion, Mach number similarity criterion, etc. The criterion is usually obtained by analysing dimensionless parameters (such as Reynolds number *Re*, Froude number *Fr*, etc.) derived from Navier–Stokes (NS) equations with the formulation analysis method or dimensional analysis method. Theoretically, with appropriate boundary conditions, satisfying all dimensionless parameters would result in sim-

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ilar flow fields. However, to the best of our knowledge, few criteria can make two flow fields strictly similar in the literature, because it is generally thought to be impossible to match all of the dimensionless parameters of two different flow fields simultaneously. For example, Wang [11] said that "*it is always impossible to meet the whole of the criteria*", and Saha [18] thought that "*only partial similarity between the model and the prototype is satisfied for most of the dimensionless parameters because usually there is conflict between the similarity requirements*". As a result, workers have relaxed some dimensionless parameters according to the specific conditions of their research projects (for instance, Mach number M could be ignored for low-speed air, because the air is almost incompressible [1]; and Re could be ignored when it exceeds a specific value [26]). Due to the neglect of some dimensionless parameters, the criteria cannot make all the parameters of two flow fields similar simultaneously. This means that when studying a phenomenon (such as thermo-convection, flow separation) that would be influenced by all the parameters of a flow field with scale models, the existing criteria would become ineffective.

The aim of this study is to examine systematically and comprehensively how to make two flow fields strictly similar. The remainder of this paper is organised as follows. In Section 2, a new similarity criterion for a gas flow field is derived from NS equations based on molecular properties (based on the variable hard sphere model assumption). Meanwhile, the similarity relationship of aerodynamic heat flux and pressure on a solid surface between two similar flow fields is deduced based on the new similarity criteria. In Sections 3 and 4, simulations are performed to validate the criterion using the CFD method. Finally in Section 5, we end with some concluding remarks.

2. Theoretical analysis

2.1. Similarity criterion for gas flow fields

In previous studies, physical properties (such as viscosity coefficient, heat transfer coefficient, etc.) are always regarded as equal, which leads to conflict between the similarity requirements. Taking the Mach number and Froude number as an example:

$$M \propto u/\sqrt{T}, \qquad Fr \propto u^2/gL$$

where g is gravitational acceleration ($\mathrm{m/s^2}$), L is characteristic length (m), T is temperature (K), u is flow velocity (m/s).

Since the property parameters are related to temperature, in order to make physical properties of two flow fields the same, the values of temperature should be the same too. Then the flow velocity ratio should be equal to 1 to make Mach numbers the same. At this time, $Fr_1/Fr_2 = gL_2/gL_1 \neq 1$, which indicates that the parameters conflict.

Based on the above analysis, in this paper, we try to derive a new similarity criterion coupled with physical properties that were obtained based on the variable hard sphere model. This model, which was put forward by Bird [27], is a binary collisions model (usually the gas molecular collision can be regarded as a binary collision). It was originally proposed for the Direct Simulation Monte Carlo method (DSMC). The DSMC method is consistent with the Boltzmann equation [28–30], and the NS equation is a first-order approximation of the Boltzmann equation [31], so that the physical properties that were obtained based on the variable hard sphere model can also be used in the NS equation [32,33]. In fact, these physical properties are in line with the real gas properties.

A similarity criterion requires geometrical similarity, kinematic similarity, dynamic similarity, thermal similarity and boundary conditions similarity. Geometrical similarity means that the ratios of all corresponding dimensions between the model and prototype are equal. Kinematic similarity means that the ratios of fluid velocities and velocity gradients at corresponding locations between the model and prototype are equal. Dynamic similarity means that the ratios of all forces at corresponding points between the model and prototype are the same. Thermal similarity, finally, means that the corresponding temperature and heat-flow fields between the model and prototype are similar.

Geometrical similarity is easy to meet, so we mainly focus on the other requirements here. To ensure the correctness of the theory, two theoretical methods are used for similarity criterion derivation: formulation analysis and dimensional analysis.

2.1.1. Formulation analysis

A. Analysis for flow field

The dimensionless Navier–Stokes equations for gas flow are given as follows:

$$\begin{cases} \dfrac{u_\infty \rho_\infty d\rho^*}{Ldt^*} + \rho_\infty \rho^* \dfrac{u_\infty \partial u_i^*}{L\partial x_i^*} = 0 \\ \rho_\infty \rho^* \dfrac{u_\infty^2 du_i^*}{Ldt^*} = \rho_\infty g \rho^* f_i^* - \dfrac{p_\infty \partial p^*}{L\partial x_i^*} \\ \quad + \dfrac{\partial}{L\partial x_j^*}\left[\mu\left(\dfrac{u_\infty \partial u_i^*}{L\partial x_j^*} + \dfrac{u_\infty \partial u_j^*}{L\partial x_i}\right) - \dfrac{2}{3}\mu\delta_{ij}\dfrac{u_\infty \partial u_k^*}{L\partial x_k^*}\right] \\ \dfrac{u_\infty d(c_p T_\infty T^*)}{Ldt^*} = \dfrac{1}{\rho_\infty \rho^*}\dfrac{p_\infty u_\infty dp^*}{Ldt^*} \\ \quad + \dfrac{1}{\rho_\infty \rho^*}\dfrac{\partial}{L\partial x_i^*}\left(K\dfrac{T_\infty \partial T^*}{L\partial x_i^*}\right) \\ \quad + \dfrac{1}{\rho_\infty \rho^*}\left(\dfrac{1}{2}\mu\left(\dfrac{u_\infty \partial u_i^*}{L\partial x_j^*} + \dfrac{u_\infty \partial u_j^*}{L\partial x_i^*}\right)^2 - \dfrac{2}{3}\mu\left(\dfrac{u_\infty \partial u_k^*}{L\partial x_k^*}\right)^2\right) \end{cases} \tag{1}$$

where p is pressure (Pa), t is time (s), f is unit mass force ($\mathrm{m/s^2}$), c_p is constant pressure specific heat (J/(kg K)), K is thermometric conductivity (W/(m k)), ρ is density ($\mathrm{kg/m^3}$), μ is viscosity coefficient (Pa s), and δ_{ij} is the Kronecker tensor. The subscript ∞ denotes incoming flow, and * denotes a dimensionless parameter.

According to kinetic theory, the viscosity coefficient [32,33] can be written as:

$$\mu = \mu_0\left(\frac{T}{T_0}\right)^\omega \tag{2}$$

where μ_0 is the viscosity coefficient at temperature T_0, and ω is viscosity temperature index.

According to Chapman–Enskog theory,

$$K = \frac{9\gamma - 5}{4\gamma - 4}\frac{k}{m}\mu \tag{3}$$

where k is the Boltzmann constant (1.38×10^{-23} J/k), and m is molecular mass (kg).

When the fluid medium can be regarded as an ideal gas, $p = \rho kT/m$. Substituting Eqs. (2) and (3) into Eq. (1), it can then be