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Void fraction influence on added mass in a bubbly flow

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ABSTRACT

This paper proposes a relation for the added mass coefficient of spherical bubbles depending on void fraction based on results obtained by a semi-analytical method.

This information is essential to completely characterize finely dispersed bubbly flows, where small spherical gas bubbles are present in a continuous liquid phase. Most of the closure relations for Euler–Euler or Euler–Lagrange models are obtained from experiments involving single bubbles. Their applicability to systems with high void fraction is therefore questionable.

This paper uses solid harmonics to solve 3D potential flow around bubbles. Several configurations were calculated for different numbers of particles and spatial configurations. Our results are compared with previous studies. Depending on the model proposed by previous authors, added mass forces could increase or decrease with void fraction. This paper solves these discrepancies by underlining the effect of induced added mass.

The main purpose of this work is to develop simple formulas fitting our semi-analytical results. These simple formulas are suitable for further use, particularly as added mass models for multiphase flow averaged equations.

1. Introduction

We are concerned with the motion of a body surrounded by a fluid. The fluid mass displaced by the body increases its inertia which defines the added mass. The added mass force acting on a body is defined as:

$$\mathbf{F_M} = -\rho V \hat{\mathbf{C}} \frac{\mathbf{dU}}{\mathbf{dt}}, \tag{1}$$

V stands for the volume of the body and ρ for the fluid density surrounding the body. Its velocity $\mathbf{U}$ is expressed in the six degrees of freedom (translation and rotation). Including rotation requires inclusion of torques in $\mathbf{F_M}$, which is therefore a 6 components vector. $\hat{\mathbf{C}}$ is a tensor with 6×6 components. If we only consider translation, $\mathbf{F_M}$ becomes a 3 component vector and $\hat{\mathbf{C}}$ a 3×3 component tensor. $\hat{\mathbf{C}}$ is called induced inertia tensor by Batchelor [1].

As a consequence of $\hat{\mathbf{C}}$, $\mathbf{F_M}$ and $\mathbf{dU}/\mathbf{dt}$ are generally misaligned depending on the body shape and the presence of other bodies or walls. This paper will only focus on the added mass coefficient of spherical bodies (bubbles).

This information is essential to completely characterize finely dispersed bubbly flows, where small spherical gas bubbles are present in a continuous liquid phase. Using either Porous Medium, Euler–Euler or Euler–Lagrange models, some authors use the widely accepted closure correlations (a.o. closure relations proposed by Tomiyama et al. [2,3]). As underlined by Darmana et al. [4], since most of the closures are empirically obtained from experiments involving single bubble, their applicability to systems with high void fraction is questionable. Moreover, even if some correlations available in the literature take into account the effect of the local void fraction on added mass, very few numerical models use them. Ishii and Hibiki [5] propose the use of correlation depending on void fraction proposed by Zuber [6] but Tomiyama et al. [2,3] propose the use of the added mass of a single bubble.

In particular, the effect of induced added mass on surrounding bubbles should be more appropriately emphasized. The induced added mass is the force exerted by one accelerating body to another through the fluid. When literature results are available, comparison with our results will be made. This paper uses solid harmonics to solve 3D potential flow around bubbles with various configurations.

A sphere in an infinite fluid medium experiences an added mass force collinear to its acceleration and the tensor $\hat{\mathbf{C}}$ is reduced to $\hat{\mathbf{C}} = 0.5\mathbf{I}$. $\mathbf{I}$ stands for the identity tensor [7,8]. This means that

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Nomenclature

Variables :

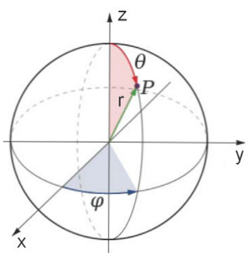
a :	bubble radius (m)
$\mathbf{F}$:	force (N)
P :	fluid pressure (Pa)
$\mathbf{U}$:	velocity (m/s)
ρ :	mass density (kg/m^3)

Subscripts:(see figure below)

r, θ, φ: Spherical coordinates
x, y, z: Cartesian coordinates

Dimensionless numbers:

ε: Void fraction
C_M: Added mass coefficient



Spherical coordinates are :
r, the radial distance
$\theta \in [0\ \pi]$, the polar angle
$\varphi \in [0\ 2\pi]$, the azimuthal angle.

Cartesian coordinates are :
$x = r\cos\varphi\sin\theta$,
$y = r\sin\varphi\sin\theta$ and
$z = r\cos\theta$

a sphere displaces a volume of surrounding fluid equivalent to half of its volume. For the sake of brevity, we will study the added mass of spherical bodies which we will designate as bubbles.

Added mass forces are of interest in naval research (inertia forces of underwater or floating objects), for the chemical industry (bubble chamber), for the energy industry (oil and nuclear) and any application involving multiphase fluid dynamics. These industrial applications are particularly affected by two factors that can strongly modify inertia forces on dispersed bubbles or particles: presence of walls and other bubbles. A simple formula for $\hat{\mathbf{C}}$, depending only on these two factors, is needed to construct a good estimation of forces acting within a multiphase mixture.

Some of the simplest situations have already been solved analytically from the perspective of potential flow theory [6,7,9–13]. First authors [6,7,9–11] accordingly with the first formulation of Zuber [6] found an increase of added mass with void fraction [13]. Wallis however found that the added mass force decreases with the void fraction [12]. Cai and Wallis proposed a more general description of added mass with two limiting cases as the one suggested by Zuber [6] corresponding to an upper bound and the one suggested by Wallis [12] corresponding to the lower bound, with an unknown parameter λ related to the external impedance of the cell around the bubble. The external impedance depends on the boundary conditions of the cell related to the bubble configuration. But as this value remains unknown, the authors concluded that there "may not exist a universal definition of added mass for an array of particles that can be applied to all the situations". Our paper solves this issue and proposes a model that can be applied for any array of identical spherical particles assuming a potential flow.

Some researchers have extended the first formulation of Zuber [6]. For example, Spelt and Sangani include velocity fluctuation effects [14] or Kushch et al. ellipsoidal bubble shape effects [15]. Some researchers have conducted DNS simulations [16–18] and finally few others have conducted experiments [19,20]. In the case of two bubbles, the numerical results show no influence of the Reynolds number (0.1–300) or of the acceleration parameter (0.1–1000) on the added mass force and a very good agreement between DNS and potential flow theory [17].

It is clear that the added mass force is a key parameter in the description of multiphase systems. It is particularly important in situations where the density ratio is large and the motion is unsteady. Thus, in this paper, we use a semi analytical method to explore the possibility of determining the added mass force in a variety of important situations.

Following the introduction, the paper includes four sections. In the first section, we expose a method to calculate the unsteady potential flow for a cloud of bubbles. The second section presents the procedure to deduce added mass force. The third section presents results for two bubbles. The fourth section presents a new formulation for the added mass forces. This formula is compared to results with a bubble close to a wall, a row and a column of bubbles. The fifth section presents results for regular and random clouds of bubbles and shows that the new model is able to accurately predict the added mass forces inside a cloud of bubbles. Finally, the main results are summarized in the conclusion section. Appendices are dedicated to readers wishing to have more information on the methodology.

2. Potential flow

2.1. Solid harmonics

If Φ is the velocity potential, it is the solution of Laplace's equation $\nabla^2\Phi = 0$ and can therefore be expressed as:

$$\Phi(\mathbf{r}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} f_\ell^m \tilde{R}_\ell^m(\mathbf{r}) + g_\ell^m \tilde{S}_\ell^m(\mathbf{r}), \tag{2}$$

where f_ℓ^m and g_ℓ^m are constants and $\tilde{R}_\ell^m$ and $\tilde{S}_\ell^m$ are respectively the normalized regular and irregular solid harmonics, solutions of Laplace's equation :

$$\begin{aligned}\tilde{R}_\ell^m(\mathbf{r}) &= \tilde{R}_\ell^m(r,\theta,\varphi) = (-1)^{\frac{|m|+m}{2}} r^\ell \tilde{Y}_\ell^m(\theta,\varphi) \quad -\ell \le m \le \ell.\\ \tilde{S}_\ell^m(\mathbf{r}) &= \tilde{S}_\ell^m(r,\theta,\varphi) = (-1)^{\frac{|m|+m}{2}} \frac{\tilde{Y}_\ell^m(\theta,\varphi)}{r^{\ell+1}} \quad -\ell \le m \le \ell.\end{aligned} \tag{3}$$

$\tilde{Y}_\ell^m(\theta,\varphi)$ are normalized spherical harmonics generally defined as

$$\tilde{Y}_\ell^m(\theta,\phi) = \sqrt{\frac{(\ell-|m|)!}{(\ell+|m|)!}}\, P_\ell^{|m|}(\cos\theta)\, e^{im\phi} \quad -\ell \le m \le \ell \tag{4}$$

where P_ℓ^m are the associated Legendre polynomials. Note that the fully normalized associated Legendre polynomials are normalized such that

$$\int_{-1}^{1}\left(\tilde{P}_\ell^m\right)^2 dx = 1 \quad 0 \le m \le \ell. \tag{5}$$

and are related to the unnormalized associated Legendre polynomials by

$$\tilde{P}_\ell^m(x) = (-1)^m\sqrt{\frac{(2\ell+1)(\ell-m)!}{2(\ell+m)!}} P_\ell^m(x) \quad 0 \le m \le \ell. \tag{6}$$

Therefore, we can also define $\tilde{Y}_\ell^m(\theta,\varphi)$ as :

$$\tilde{Y}_\ell^m(\theta,\phi) = (-1)^m\sqrt{\frac{2}{2\ell+1}}\tilde{P}_\ell^{|m|}(\cos\theta)\, e^{im\phi} \quad -\ell \le m \le \ell. \tag{7}$$

For an isolated sphere of radius a_n and velocity $(U_n, \theta_n = 0, \varphi_n = 0)$ expressed in spherical coordinates, the velocity potential is well known [7] and defined as :

$$\phi_n = -\frac{U_n a_n^3}{2}\frac{\cos\theta}{r^2} = -\frac{U_n a_n^3}{2}\frac{z}{r^3}. \tag{8}$$

Applying a rotation of angle θ_n around the y-axis and angle φ_n around the z-axis, the general expression for the potential with the velocity expressed in spherical coordinates $(U_n, \theta_n, \varphi_n)$ is deduced:

$$\phi_n(\mathbf{r}) = -\frac{U_n a_n^3}{2r^2} \times \left[\cos\theta_n\cos\theta + \sin\theta_n\sin\theta\,(\sin\varphi_n\sin\varphi + \cos\varphi_n\cos\varphi)\right]. \tag{9}$$

Introducing $\dot{x}_n, \dot{y}_n, \dot{z}_n$ we have:

$$\phi_n(\mathbf{r}) = -\frac{a_n^3}{2r^2}\left(\dot{z}_n\cos\theta + \dot{x}_n\sin\theta\sin\varphi + \dot{y}_n\sin\theta\cos\varphi\right) \tag{10}$$