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Bifurcations and eddy genesis of Stokes flow within a sectorial cavity PART II: Co-moving lids

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HIGHLIGHTS

- We considered the problem of the flow in a sectorial driven cavity.
- Analytical method is used to solve the problem.
- The effects of the cavity aspect ratio and the speeds ratio are analyzed.
- The bifurcation diagrams and the eddy genesis mechanism are given.
- The general concepts for location and classification of stagnation points are presented.

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ABSTRACT

This paper is a complementary study to earlier work (Gürcan, 2013), concerning eddy genesis and bifurcations in a sectorial driven cavity for Stokes flow, consisting of a pair of curved stationary side-walls and two straight translating lids moving in the same radial direction with speeds U_1 and U_2. The flow is governed by two physical control parameters: the cavity aspect ratio $A = \frac{r_2}{r_1}$ where r_1 and r_2 are the radii of the inner and outer curved side-walls, respectively and the ratio of the upper and lower lid speeds ($S = \frac{U_1}{U_2}$). In previous work (Gürcan, 2013), focus was on lids moving in the opposite direction or a single moving lid.

By considering different A values for each S, the streamline patterns and their bifurcations are studied; occurrence of stagnation point bifurcations and transformation of the local flow topology are observed at critical values of A and S. The key results are shown in an (S, A) control space diagram which exhibits an intricate structure due to the intersection and confluence of eight bifurcation curves representing flow bifurcations at degenerate critical points. Six different eddy genesis mechanisms are observed for $S \in (0, 1]$ and $A \in (1.45, 20)$. In each case the flow structures and eddy genesis mechanisms are illustrated in detail with figures and bifurcation diagrams. The study also serves as a benchmark problem for testing new numerical algorithms.

1. Introduction

Lid-driven cavity flow occurs in many engineering applications such as roll coating [1], polymer processing [2] and ceramics casting [3], enhancing mass transport [4] and global heat transfer via thermal mixing [5]. Due to its simple geometry, this type of flow is important not only in its own right as a basic physical model but also serves as a potential benchmark problem for testing new numerical algorithms.

There are many studies in the literature on 2-D cavity flows related to eddy structure, bifurcations and streamline topologies. One of the most important of these is that of Gaskell et al. [6] for flow in a half-filled annulus, in which a boundary value problem for the streamfunction, ψ is given as a function of the radius aspect ratio and speed radio. Their study focused on both the counter-rotating ($S < 0$) and co-rotating ($S > 0$) cases, showing that the fluid flow, according to these cases, exhibited one and two main eddies, respectively. By varying R and S, they found the changes in flow structure to be due to stagnation point bifurcations in which a center is transformed into a saddle point and vice versa.

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Notation

r_1, r_2	Radius of inner and outer circles respectively
A	Cavity aspect ratio $= r_2/r_1$
2α	Angle of the sector
E_n, F_n	Streamfunction expansion coefficients
n	Summation index
λ_n	Streamfunction expansion eigenvalues
$\underline{u}$	Dimensionless fluid velocity
S	Speed ratio of moving lids $= U_1/U_2$
U_1, U_2	Top and bottom lid speeds respectively
ϕ_1^n	Streamfunction expansion eigenfunctions
ψ	Streamfunction

The generation of eddies in a rectangular cavity was studied by Gürcan et al. [7] with increasing A for different speed ratios, S, of the moving lids ($S < 0$). To construct a control space diagram, a particular region of (S, A) parameter space (i.e. $-1 \leq S < 0$ and $0 < A < 3.2$) was considered. In as subsequent paper, [8], the eddy generation process in deep cavities, where both negative and positive speed ratios are considered, was explored. This work lead to an extended control space diagram for Stokes flow in a rectangular cavity. Gürcan and Deliceoğlu [9] investigated streamline patterns and their bifurcations for Stokes flow in a rectangular cavity having two fixed walls and two moving lids for different values of the cavity aspect ratios by using the normal-form transformations.

Flow bifurcation and eddy generation for steady, viscous flow in an L-shaped cavity, with the lids moving in opposite directions, has been investigated by Deliceoğlu and Aydın [10]. They obtained new bifurcation curves and flow patterns via the generation of a separatrix.

Sznitko and Zielinski [11] investigated the transitional flow in a co-and counter-rotating annular cavity, of aspect ratio $A = 5$, using the three-dimensional DNS method. They analyzed the spatial structures which appear in the boundary layers of both disks as well as the time dependence of these flows.

Recently, Gürcan and Bilgil [12] analyzed the generation of eddies in a sectorial cavity with decreasing A for different speed ratios of the moving lids ($S \leq 0$). A particular region of parameter space (S, A) where $A \in (1.6, 6.5)$ and $S \in [-1, 0]$, was considered to construct a control-space diagram exhibiting several critical curves representing flow bifurcations at degenerate critical points. When the aspect ratio of the cavity changes, the number and structure of the primary eddies change strongly, e.g. if the aspect ratio increases then the number of eddies also increase in the cavities ($A \to 0$, i.e., when $r_2 \to r_1$). This situation can be found in the Refs. [13–17].

Cavities with curved surfaces such as a sectorial cavity may occur naturally or as a necessity of a particular manufacturing process [18]. In industrial devices, the mixing problem of viscous fluid flows in several kinds of two-dimensional cavity is presented by Woering et al., [19].

In this paper, we investigated the Stokes flow in a sectorial cavity (for $S > 0$). The biorthogonality conditions for this problem was considered by Khuri [20]. Although the solution for Stokes flow in a single lid-driven sectorial cavity was given in this paper, he was not interested in the degenerate critical points and flow bifurcations. Here, we focus on the eddy structure, bifurcations and streamline topology by using the approaches of Gürcan et al. [8] and Gaskell et al. [6] to study the effects of cavity aspect ratio, speed ratio, and curvature on the structure of the flow. Although the annular cavity flows in the literature show geometrical similarity with our cavity, the moving lids in their cavity are curve edges in general while the curve edges are fixed and the flat plates give the movement to the fluid in this study. Therefore the flow structures and the bifurcations are different from the annular cavity flows, in our problem. This work is an extension of previous work [12], which to our knowledge, is the first such study in the literature for the case of both upper and lower lids moving in the same radial direction (co-moving).

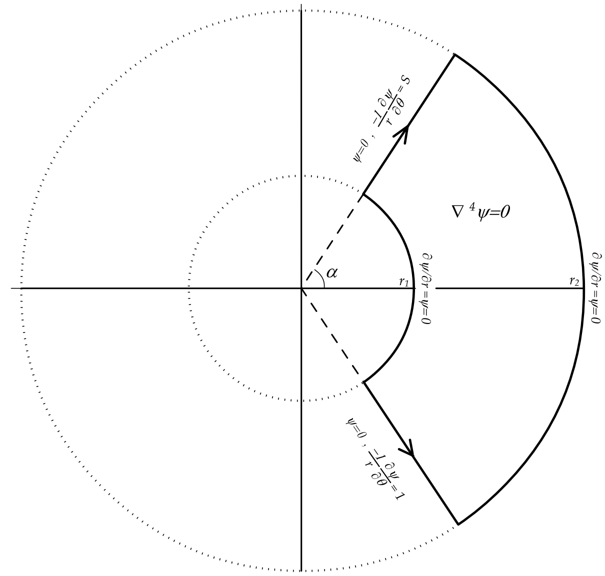

Fig. 1. Geometry and boundary conditions for a lid driven sectorial cavity two solid stationary curved side walls and driven by upper and lower enclosing lids.

2. Problem specification and solution

2.1. Flow geometry, field equations and boundary conditions

A two-dimensional sectorial cavity $\nu = \{(r, \theta) : r_1 \leq r \leq r_2, -\alpha \leq \theta \leq \alpha\}$ is filled with incompressible fluid (Fig. 1). The boundaries $\theta = \alpha$ and $\theta = -\alpha$ are two flat plates, or lids, which translate with speeds U_1 and U_2 in the radial direction (we fix $U_1 = S$ and $U_2 = 1$), thus setting the fluid into motion. Speeds are non-dimensionalized with respect to U_2 and specifically $\alpha = \frac{\pi}{4}$ such that the flow domain is given by:

$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}, \quad r_1 \leq r \leq r_2. \tag{2.1}$$

The two-dimensional, steady, isothermal flow of a Newtonian, incompressible fluid of constant density ρ and viscosity η is governed by the Navier–Stokes and continuity equations written in non-dimensional form as:

$$\mathrm{Re}\, \underline{u} \cdot \nabla \underline{u} = -\nabla p + \nabla^2 \underline{u}, \quad 0 = \nabla \cdot \underline{u}, \tag{2.2}$$

where $\mathrm{Re} = \rho U_2 A/\eta$ is the Reynolds number measuring the relative importance of inertia to viscous forces, which under Stokes flow conditions (i.e. in the absence of inertia) is set to zero. Taking the *curl* of the remaining terms to eliminate the pressure and introducing the streamfunction $\psi(r, \theta)$, via the continuity equation, yields the biharmonic equation:

$$\nabla^4 \psi(r, \theta) = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \right)^2 \psi(r, \theta) = 0. \tag{2.3}$$

The derivatives of ψ give the velocity components:

$$u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = \frac{\partial \psi}{\partial r}, \tag{2.4}$$

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