European Journal of Mechanics B/Fluids 55 (2016) 246–258

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Equilibration of centrifugally unstable vortices: A review

G.F. Carnevale [a,*], R.C. Kloosterziel [b,c], P. Orlandi [d]

[a] Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA 92093, USA
[b] School of Ocean and Earth Science and Technology, University of Hawaii, Honolulu, HI 96822, USA
[c] Royal Netherlands Institute for Sea Research, PO Box 59, 1790 AB Texel, The Netherlands
[d] Dipartimento di Meccanica e Aeronautica, University of Rome, "La Sapienza", via Eudossiana 18, 00184 Roma, Italy

ARTICLE INFO

Article history:
Available online 16 September 2015

Keywords:
Vortex dynamics
Stability
Rotating flow

ABSTRACT

In three-dimensional flow, a vortex can become turbulent and be destroyed through a variety of instabilities. In rotating flow, however, the result of the breakup of a vortex is usually a state comprising several vortices with their axes aligned along the ambient rotation direction. This article is a review of our recent work on how the combined effect of centrifugal and barotropic instabilities can breakup a vortex and lead to its reformation in a predictable way even though an intermediate stage in the evolution is turbulent. Centrifugal instability tends to force the unstable vortex into a turbulent state that mixes absolute angular momentum in such a way as to precondition the flow for a subsequent barotropic instability. A method for predicting the redistribution of angular momentum and the resulting velocity profile is discussed. The barotropic instability horizontally redistributes the component of vorticity that is aligned along the ambient rotation vector, resulting in the final byproducts of the instability, which are stabilized by the effects of ambient rotation. A prediction scheme that puts the tendencies of these two instabilities together proves to be very reliable.

Contents

1. Introduction

Vortices and currents in the oceans are subject to a variety of important instabilities including centrifugal, barotropic and baroclinic instabilities. One of our goals has been to achieve an understanding of how these instabilities participate in the formation and maintenance of observed vortices and currents. In experiments, both laboratory and numerical, which can be designed such that only centrifugal and barotropic instability are relevant, a great deal of insight has been achieved, and it is now possible to predict, to a great extent, the results of the combined effects of centrifugal and barotropic instabilities. The purpose of this article is to review this development in the case of anticyclonic vortices in some detail. The cases of unstable cyclonic vortices and planar currents have also been dealt with rather successfully, and we shall point the reader to the appropriate literature for those results.

Our motivation for a detailed study of the evolution of centrifugal instability in the case of anticyclonic vortices came from the laboratory studies discussed in [1]. The basic experiment is illustrated schematically in Fig. 1. The experiments were performed in

* Corresponding author.
E-mail address: gcarnevale@ucsd.edu (G.F. Carnevale).

http://dx.doi.org/10.1016/j.euromechflu.2015.06.007

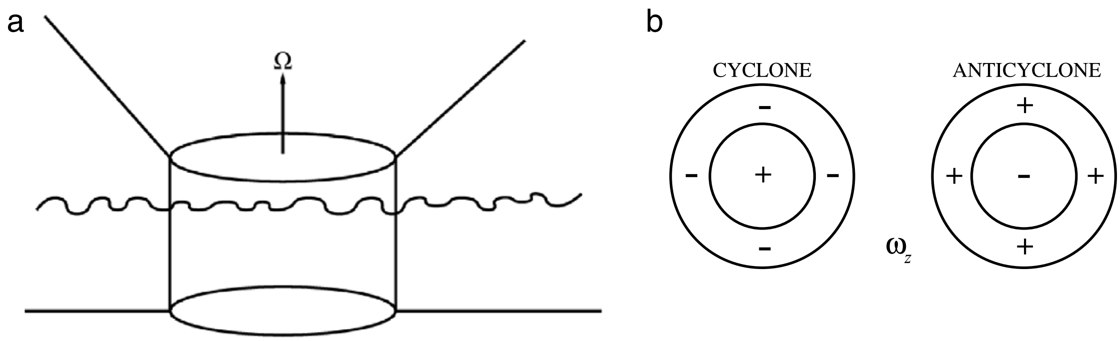

Fig. 1. Schematics illustrating the basic laboratory experiments investigated in [1]. (a) A hollow cylinder is placed vertically within a tank rotating about a vertical axis. The tank is filled with water to a level which is lower than the height of the inserted hollow cylinder. The flow is stirred either cyclonically or anticyclonically within the hollow cylinder and allowed to adjust to a smooth flow, at which point the hollow cylinder is lifted vertically by the agency of the supports attached to its upper rim. (b) The vertical vorticity field within the hollow cylinder just before it is removed from the tank is either that of a cyclone or an anticyclone. Since these are isolated vortices with no flow initially outside the cylinder, they take on the form of a core of cyclonic/anticyclonic vorticity surrounded by an annulus of vorticity of the opposite sign. The total vorticity within the cylinder must vanish since the velocity on the cylinder is zero.

a tank rotating about a vertical axis with angular velocity Ω and containing a layer of water. A hollow cylinder, of height more than the depth of the water, was inserted into the tank and left standing vertically. A vortex was then created by stirring within the cylinder either cyclonically (in the same direction of the rotation of the tank) or anticyclonically. The cylinder was lifted vertically, leaving behind a vortex in the quiescent surrounding fluid. In the case of cyclonic vortices, the subsequent evolution was relatively smooth, more or less two-dimensional, that is independent of depth. As shown in Fig. 2, in this case the vortex typically evolved into a cyclonic vortex with two satellite anticyclonic vortices, one on either side of the core cyclone. This configuration is referred to as a tripole (see e.g. [2,3]). In the cases in which the initial vortex within the cylinder was an anticyclone, the release was followed by a brief period of vigorous turbulent motion throughout the vortex column followed by the vortex being torn apart resulting in two dipolar vortices that moved away from each other as they became more and more two-dimensional [1]. An example of this type of evolution is shown in Fig. 3.

If the flow in these experiments were at all times two-dimensional, then except for the sign of the velocity field, one would expect that the behavior would be the same for cyclones and anticyclones, for then there is nothing in the physics that would break the (anti-)symmetry of the flows. This led Kloosterziel and van Heijst [1] to conclude that the early three-dimensional turbulent motion so prominent in the anticyclonic case leads to the difference between the evolution of cyclones and anticyclones. At the moderate initial velocities in these experiments, one would expect anticyclones but not cyclones to be vulnerable to centrifugal instability. Thus, Kloosterziel and van Heijst argued that it was centrifugal instability that somehow changed the vorticity distribution in such a way as to make the results very different in the two cases.

The redistribution of vorticity from a monopolar form as shown in Fig. 1(b) into forms such as dipoles or tripoles as seen in the laboratory experiments can be accomplished by barotropic instability. Such an instability can produce a variety of complex forms, most of which are unstable and typically degenerate into monopoles, dipoles and tripoles. We investigated this in detail in Carnevale and Kloosterziel [4]. Indeed, one of these intermediate complex forms, a quadrapole, was already observed in the laboratory by Kloosterziel and van Heijst [1]. Additionally, theoretically it should have been possible even for an anticyclone to become a tripole. In fact, we found an example of this in later laboratory experiments [5] as shown here in Fig. 4, but this was a rare event.

Through three-dimensional numerical simulations, Orlandi and Carnevale [6] investigated the dual role played by barotropic and centrifugal instabilities. At high Reynolds numbers, simulations with no ambient rotation show that a perturbed vortex column will just degenerate into 3D turbulence (see for example Carnevale et al. [7]). With ambient rotation, however, the flow will eventually tend to columnar (i.e. vertically uniform) flow, in accord with the Taylor–Proudman theorem. Thus it is not a surprise that after the initial rather turbulent phase of the anticyclonic vortex, the flow becomes two-dimensional (columnar). Orlandi and Carnevale [6] confirmed Kloosterziel and van Heijst's [1] hypothesis that the centrifugal instability set up the conditions for the barotropic instability to create a pair of dipoles in the anticyclonic case rather than a compact tripole as in the cyclonic case. Centrifugal instability was shown to increase the horizontal velocity gradients in the flow and this triggered the barotropic instability that led to double dipoles. The precise nature and effect of the centrifugal instability was shown to depend on the details of the initial distribution of vorticity. The possible evolution of an anticyclone into a tripole as well as double dipoles after a centrifugal instability was also confirmed. Additionally, the intermediate form a quadrapole was also produced.

Thus we had shown that 3D numerical simulation could capture all of the phenomena that had been observed in the laboratory. This was not entirely satisfying because it was not yet possible to predict, from a knowledge of only the initial state, what structures, intermediate and final, would emerge from an initially centrifugally unstable vortex. We proceeded to make a thorough study of centrifugal instability. In [8], we found that in the limit of infinite Reynolds number, the distribution of vorticity could be predicted precisely knowing only the initial distribution. This was the key to predicting the combined effect of both centrifugal and barotropic instabilities. We were then able to devise a new scheme that allowed us to predict the evolution of the full 3D flow based solely on this new knowledge of the precise effect of centrifugal instability and the two-dimensional evolution of barotropic instability.

In what follows, we shall explain how this prediction scheme works and show evidence for its effectiveness. In Section 2, we review the necessary background information about the barotropic instability. In Section 3, we discuss the method of predicting the precise result of centrifugal instability in the limit of infinite Reynolds number. In Section 4, we show how this new knowledge of how centrifugal instability equilibrates can be used in the prediction of the outcome of the combined centrifugal and barotropic instabilities. In Section 5, we conclude with a discussion of how the methods discussed here have been extended to initially planar barotropic currents.