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The onset of recirculation flow in periodic capillaries: Geometric effects

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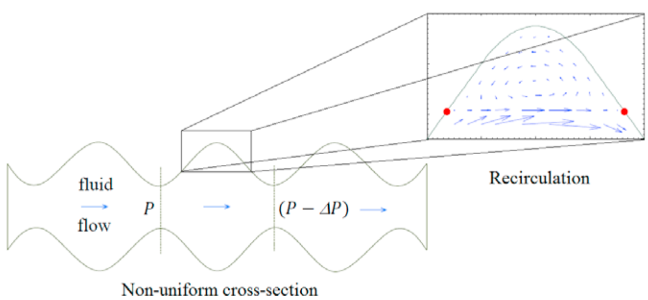
HIGHLIGHTS

- Creeping fluid flow in tubes of periodically varying cross-section is investigated.
- Full BEM solution of Stokes' equations is formulated for an infinite periodic tube.
- Recirculation flow occurs above a critical amplitude dependent on tube geometry.
- Two recirculation zones were found for large amplitudes in all capillaries studied.
- A numerical study characterises the onset of first and second order recirculation.

GRAPHICAL ABSTRACT



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ABSTRACT

In this paper we examine the onset of flow circulation in expansion regions of infinite tubes of periodic, non-constant cross-section. Three types of axisymmetric capillary shapes were considered; sinusoidal, parabolic and triangular. A full boundary element method (BEM) solution of Stokes' equations was formulated for the specific case of an infinite periodic tube. Geometric parameters were varied to establish conditions for the onset of recirculation. Recirculation flow is first predicted to appear beyond a critical amplitude, for all types of tubes studied, with zones in tubes of triangular sections appearing at a lower amplitude. Second order recirculation zones were predicted for still higher amplitudes, in all the capillaries. A numerical study was undertaken to characterise the onset of first and second order recirculation flows in terms of geometric factors.

1. Introduction

Flow through an axisymmetric pipe of non-uniform cross-section has been a topic of interest in fluid dynamics for some time [1–7]. More recently, this interest has been renewed through recognition of its potential to separate particle dispersions within microfluidic devices and nanoporous membranes [8,9]. Other applications include the investigation of transport processes in porous media [10] and biomechanics (e.g. blood flow through arteries [11] or the transport of intestinal fluid through the colon [12]).

In this paper, we investigate the effect of tube shape and geometry on the flow characteristics within the tube. Under certain conditions [13] (dependent on both the geometric properties of the tube and the physical properties of the fluid), it is possible to observe detachment of the flow from the pipe wall near the

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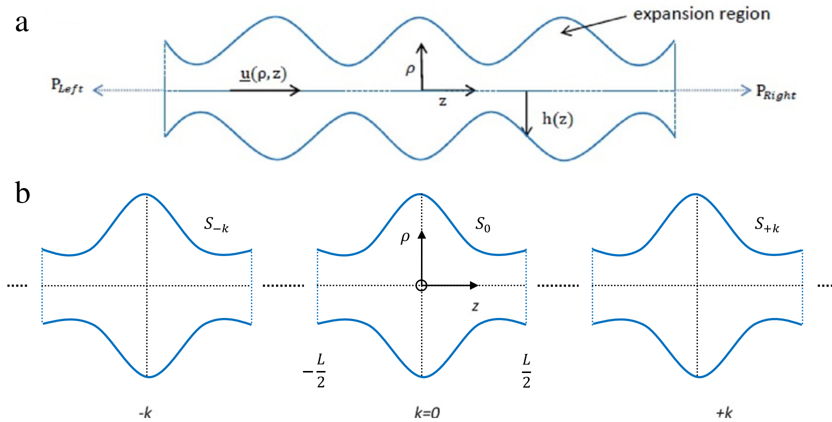

Fig. 1. (a) Schematic of an infinitely long, cylindrical periodically constricted capillary, $\bar{h}(\bar{z}) = \bar{h}(\bar{z} + nL)$, shown in dimensional variables. (b) Geometrical configuration of a periodic tube, where z and r are the axial and radial directions, respectively showing the zeroth wave-section and the $-k$'th and $+k$'th wave-sections.

expansion regions (regions of maximum profile amplitude, see Fig. 1(a)) of the tube. In this case, recirculation regions develop in the sections of the tube where the diameter is greatest. It is the effect of the tube shape and geometry on the development of these recirculation zones which we have focused on in this paper.

Flow through tubes with sinusoidally varying cross-section has been studied by a number of groups. Chow and Soda [1] and Sisavath et al. [2] have found asymptotic series solutions for the flow in tubes whose cross-section varies slowly along its length. A number of numerical techniques, including spectral methods [3], iterative methods [4], and finite difference techniques [5] have also been used to solve the Navier–Stokes equations for flow in periodic tubes. The creeping flow problem has also been examined using collocation methods [10,6] and the boundary element method [7]. Flow of non-Newtonian fluids through pipes of varying cross-section has also been examined (see for example [11,14–16]). All publications cited above have examined the case of a sinusoidally varying axisymmetric tube with circular cross-section, and have largely focused on plotting streamlines and velocity profiles for the flow.

With the development of sophisticated manufacturing techniques [17], the production of tubes with various geometric profiles has become feasible. Despite the fact that much of the theoretical work has concentrated on sinusoidally varying tubes, these are not the easiest to manufacture. Indeed, triangular, parabolic, saw-tooth and square profiles are more likely candidates for application. Hence, it is of interest to investigate the relative differences in flow behaviour due to geometric features other than those inherent in simple wave-like profiles. To date, there has been no such comparison of different types of profiles.

In this paper, we provide a systematic study of the effect of various geometric parameters on the facilitation of the development of recirculation zones within a periodically varying tube. Three different tube profile types are considered—cosine, piecewise parabolic and triangular. We consider the creeping flow case and pay particular attention to the effect of varying the amplitude and wavelength of corrugation, the length of the expansion regions and the minimum (or throat) radius.

2. Governing equations

Consider the flow of an incompressible Newtonian fluid through an infinite axisymmetric periodic tube at low Reynolds number, see Fig. 1. An infinite periodic tube (rather than one representative wave-section) is considered to enable the specification of a pressure difference boundary condition (this is discussed in more detail in Section 3). The flow is driven by a pressure gradient $\Delta P/L$, where ΔP is the pressure drop across one wavelength of the tube and L is the wavelength of one section of the tube. The surface of the tube is $\mathbf{y} = \bar{z}\hat{\mathbf{z}} + \bar{h}(\bar{z})\hat{\mathbf{r}}$, where $\hat{\mathbf{z}}$ and $\hat{\mathbf{r}}$ are unit vectors in the longitudinal and radial directions respectively and $\bar{h}(\bar{z})$ defines the tube surface.

The equations governing the flow field in an axisymmetric tube (Fig. 1(a)) in the absence of inertial effects (small Reynolds number, Stokes flow) can be written in dimensional variables as

$$\bar{\nabla}^2\bar{\mathbf{u}} = \frac{1}{\mu}\bar{\nabla}\bar{p}, \qquad \bar{\nabla}\cdot\bar{\mathbf{u}} = 0$$

subject to no slip boundary conditions on the tube surface, S,

$$\bar{\mathbf{u}}(\bar{\mathbf{x}}) = 0, \quad \bar{\mathbf{x}} \in S$$

and pressure difference ΔP between the ends of one wavesection of the tube. Here $\bar{p}(\bar{z}, \bar{r})$ is the pressure, μ is the viscosity of the fluid and $\bar{\mathbf{u}}(\bar{z}, \bar{r})$ is the flow velocity.

On introducing the nondimensional variables, $z = \bar{z}/L$, $r = \bar{r}/L$, $p = \bar{p}/\Delta P$, $\mathbf{u} = \bar{\mathbf{u}}\mu/(L\Delta P)$, the Stokes equations for viscous flow may be written

$$\nabla^2\mathbf{u} = \nabla p, \qquad \nabla\cdot\mathbf{u} = 0.$$

The fundamental solution of the nondimensional Stokes equations is [18]

$$\begin{aligned}\mathbf{u}(\mathbf{x}) \; = \; & \frac{1}{4\pi}\int_S dS(\mathbf{y})\; \mathbf{G}(\mathbf{x},\mathbf{y})\cdot\mathbf{F}(\mathbf{y}) \\ & - \frac{1}{4\pi}\int_S dS(\mathbf{y})\; \mathbf{H}(\mathbf{x},\mathbf{y})\cdot\mathbf{u}(\mathbf{y}) \quad \text{for } \mathbf{x}\in S,\end{aligned} \qquad (1)$$

$$\begin{aligned}\mathbf{u}(\mathbf{x}) \; = \; & \frac{1}{8\pi}\int_S dS(\mathbf{y})\; \mathbf{G}(\mathbf{x},\mathbf{y})\cdot\mathbf{F}(\mathbf{y}) \\ & - \frac{1}{8\pi}\int_S dS(\mathbf{y})\; \mathbf{H}(\mathbf{x},\mathbf{y})\cdot\mathbf{u}(\mathbf{y}) \quad \text{for } \mathbf{x}\in V\end{aligned} \qquad (2)$$

on the tube surface and interior respectively. Here, dS is the surface area element of the boundary S at $\mathbf{y}$ and V is the interior region of the tube. In addition, $\mathbf{F}(\mathbf{y}) = -\boldsymbol{\Sigma}(\mathbf{y})\cdot\hat{\mathbf{n}}(\mathbf{y})$ is the force per unit area exerted on the fluid by the boundary at $\mathbf{y}$ (boundary force), where $\boldsymbol{\Sigma}(\mathbf{y})$ is the stress tensor, defined as

$$\Sigma_{ij} = -p\,\delta_{ij} + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and the normal vector $\hat{\mathbf{n}}(\mathbf{y}) = (\hat{n}_z, \hat{n}_r)$ is directed outward from the control volume V. Also, $\mathbf{G}(\mathbf{x}, \mathbf{y})$ and $\mathbf{H}(\mathbf{x}, \mathbf{y})$ are known functions of the sample point $\mathbf{x}$ and source point $\mathbf{y}$, defined as