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Analytical solutions of single and multi-phase models for the condensation of nanofluid film flow and heat transfer

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ABSTRACT

Classical Nusselt's condensate falling film theory is extended in this paper to the case when the base fluid is added ingredients of some frequently used popular nanoparticles. The resulting mixture, i.e, nanofluids, is analytically investigated either when the nanoparticles are uniformly distributed across the condensate boundary layer which is the most used model (single phase) in the literature, or when the concentration of nanoparticles through the film is allowed to vary from the wall to the outer edge of the condensate film in the light of modified Buongiorno's nanofluid model (multi-phase) incorporating mechanisms of the Brownian and thermophoretic diffusion. In both theoretical cases, momentum and energy equations are solved analytically to deduce the flow and heat transport phenomena. As a result, the influences of employed nanofluids on the flow and heat of the condensate film are determined exactly. When the concentration of nanoparticles is assumed constant both models are shown to coincide. Otherwise, effects of nanofluids as compared to the regular fluid on the velocity profiles, the mass flow rate, the thickness of the condensate film and the Nusselt number are easy to conceive from both single and multi-phase models. In particular, the theoretical treatment in both models enables us to understand the heat transfer enhancement feature of the nanofluids models. When the diffusion parameter is increased in the multi-phase model, more enhancement in the rate of heat transfer is observed. In agreement with the experimental evidences, the water-based nanofluid with nanoparticles *Ag* is the best heat transferring mixture.

1. Introduction

The heat and mass transfer processes of falling film condensation over a vertical wall have attracted many researchers after the pioneering cognitive work of Nusselt [1]. It is quite significant since condensation phenomenon has application fields in air cooled power station condensers [2], in biotechnology [3], in food processing [4] and in industrial machines [5]. Ag, Cu, Cuo, Al_2O_3 and TiO_2 nanoparticles and their effects on the aforementioned technological applications concerning the momentum and thermal transport events are investigated in the current work taking into account both single and multi-phase scenarios.

Although Nusselt's mathematical model [1] is simplified to approximately evaluate the heat transfer and fluid flow of the condensate film, it was found moderately accurate, physically tractable and later improved within the history. Since then, a large number of experimental and theoretical research were conducted by considering different flow and surface conditions [6]. Du and Zhao [7] calculated the average heat transfer coefficient of steam condensate inside a triangular channel. Wang and Rose [8] analyzed the film condensation in horizontal microchannel. The film thickness, the condensation mass flux flow and the velocity were determined from the modified equations on a vertical microchannel in [9]. Laminar falling film condensations over a vertical plate with an accelerating vapor flow were analyzed in [10] in the presence of condensate suction or slip effects at the plate surface. Slip-driven influences were examined in [11]. Local and average heat transfer behavior for a falling film on horizontal flat tubes was explored through an experimental approach in [12]. The interfacial slip in the presence of non condensable species in the bulk mixture of vapor on heat transfer characteristics in film condensation over horizontal tubes with varying radius of curvature was investigated in the recent work by Pati et al. [13].

Nanofluids entail unique properties, such as high thermal conductivity and low susceptibility to sedimentation, fouling, erosion and clogging. They are primarily used as coolant in heat transfer equipment as in heat exchangers, in electronic cooling system (flat plate and radiators [14]) and in solar collectors [15]. Colloidal suspensions of nanoparticles in a base fluid, such as water, are the main ingredients of nanofluids [16]. The nanoparticles used in

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nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil [17]. Nanofluids exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid [18]. Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids were computed in [19]. A dozen of engineering applications were given in the review paper by Buschmann [20].

Not much research has been fulfilled for the condensation film flow in the presence of nanofluids. In the experimental work of Huminic and Huminic [21], heat transfer due to the nanoparticles was observed to enhance in closed thermosiphons. Flow and temperature in a boundary layer over a flat plate were studied in [22] by considering a variable nanoparticle concentration. Such analysis was also conducted in [23] to measure the thermophoresis and Brownian motion effects in nanofluid heat transfer enhancement. During the preparation of this work, heat transfer at film condensation of stationary vapor with nanoparticles near a vertical plate was published by Avramenko et al. [24], but with constant viscosity and thermal conductivity as seen in the model equations (1)–(2) in [24]. The motivation of the present work is twofold. First, it is aimed to extend the classical theory of Nusselt of regular film condensate by adding nanoparticles like Ag, Cu, CuO, Al_2O_3 and TiO_2, while all fluid properties being dependent on the constant concentration of nanoparticles, that is the nanoparticles are uniformly and constantly distributed across the boundary layer within the single phase analysis. Laminar convective heat and mass transfer analysis is analytically explored in this case and it is shown that the presence of nanofluid enhances heat transfer rate by decreasing the thickness of film condensate. Second, the classical model of Nusselt is assessed when the nanoparticles concentration is varying from the wall to the outer edge of the film thickness in the framework of modified Buongiorno's nanofluid model [16] within the multi-phase analysis. Hence, the effects of thermophoretic and Brownian diffusions are incorporated into the flow motion. It is proved that in the absence of such diffusion mechanisms together with a constant nanoparticle concentration, the second approach is exactly matched with the first approach in line with the physical expectation. Otherwise, the momentum and thermal layers are found to be highly affected and heat transfer is much more enhanced by increasing the diffusion parameter in the multi-phase model as compared to the single phase model.

2. Mathematical formulation

The simplified model of condensation process is based on Wilhelm Nusselt's notion, which states that in a condensate film the only resistance takes place for the removal of heat in the process of condensation. Hence, condensation occurs when steam is cooled below its saturation temperature. The cooled wall transfers the heat of evaporation released during condensation.

According to the physical layout in Fig. 1, vapor is condensing on a vertical wall of temperature T_w which is lower than the saturation temperature T_v. The usual assumptions as in Nusselt's theory [1] are adopted. In order to enable the boundary layer approximation, it is further assumed that the thickness of the film is small. So, a condensate film develops flowing downwards under the influence of gravity g. As condensation occurs over the wall surface the thickness $\delta(x)$ of the film increases and the density of vapor is negligible.

2.1. Model 1: single phase

In the presence of equally distributed nanoparticles concentration, the problem is modeled either by applying the Navier–Stokes equation or directly by a force balance for a fluid element in the nano film. The fluid is a water based nanofluid containing five distinct types of nanoparticles: Ag, Cu, Cuo, Al_2O_3 and TiO_2, whose thermo physical properties are outlined in Table 1.

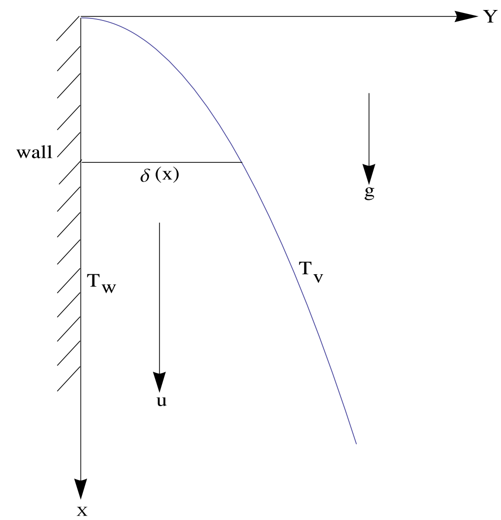

Fig. 1. Basic nanofluid film condensation configuration.

Table 1
Thermo-physical properties of water and nanoparticles [25,18,19].

	ρ (kg/m^3)	C_p (J/kg K)	k (W/mK)	$\beta \times 10^5$ (K^{-1})
Pure water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67
Copper oxide (CuO)	6320	531.8	76.5	1.80
Silver (Ag)	10 500	235	429	1.89
Alumina (Al_2O_3)	3970	765	40	0.85
Titanium Oxide (TiO_2)	4250	686.2	8.9538	0.9

Together with the energy equation, the full mathematical model is given in the thin condensate nano film by the single phase system

$$\mu_{nf} u'' = -\rho_{nf} g, \qquad u(0) = 0, \qquad u'(\delta) = 0,$$
$$T'' = 0, \qquad T(0) = T_w, \qquad T(\delta) = T_v, \tag{1}$$

where u is the velocity along the x-direction, a dash means derivative along the transverse direction y. The no-slip at the wall and the no shear at the liquid–vapor interface of the flow conditions are employed, together with uniform temperatures at both ends. The subscript nf stands for the nanofluid property, and additionally the parameters appearing in (1) are respectively,

$$\mu_{nf} = (1-\varphi)^{-2.5}\mu_f,$$
$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s,$$
$$k_{nf} = k_f \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}, \tag{2}$$

where φ is the solid volume fraction or concentration, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, k_f and k_s are the thermal conductivities of the base fluid and nanoparticle, respectively. It should be reminded that the effective thermal conductivity of the nanofluid k_{nf} approximated as above is due to Maxwell–Garnett [25]. We choose this universally acknowledged model, though there are thermal conductivities which show the effect of particle size, temperature and others on the thermal conductivity and in turn on heat transfer as can be inferred from the review papers [26–29].

After the transformations

$$u = \frac{\rho_{nf} g \delta^2}{\mu_{nf}} U, \qquad \theta = \frac{T - T_w}{\Delta T}, \qquad \eta = \frac{y}{\delta}, \tag{3}$$