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Comparison of three simple models of Kelvin's ship wake

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ABSTRACT

A theoretical explanation of observations of high-Froude-number ship wakes that are narrower than the classical Kelvin 39° angle was recently offered by Rabaud and Moisy. The explanation relies on the assumption that a ship hull does not create waves longer than its length. A validation of this theoretical model has also been given. The validation is based on the approximation of the flow created by a ship hull by means of a Gaussian distribution of pressure at the free surface. These two flow models predict a wake angle ψ_{max} that decreases like $1/F$ as the Froude number F increases beyond $F \approx 0.5$. A third theoretical explanation was recently proposed by the authors. This theoretical explanation assumes that the wave pattern of a ship mostly consists of dominant waves that are created by the ship bow and stern, and is mostly determined by interference effects between these dominant waves. The analysis of interference effects on the Kelvin wake of a ship predicts a wake angle $\psi_{max} \approx 0.14/F^2$ for a monohull ship, or $\psi_{max} \approx 0.2\sqrt{b}/F$ for a catamaran with beam/length ratio b. The 'flow models' underlying these three alternative theoretical explanations of narrow ship wakes are examined, and the corresponding theoretical predictions are compared to the 37 observations of ship wakes reported by Rabaud and Moisy for Froude numbers F within the wide range $0.1 < F < 1.7$. The wake observations are found to be consistent with the predictions given by an analysis of interference between the bow and stern waves of a monohull ship, or a catamaran with beam/length ratio b within the range $0.4 \leq b \leq 0.8$. Indeed, agreement is consistently strong for the 35 wake observations within the range $0.1 < F < 1.4$. This range of Froude numbers includes the range $F < 0.6$, where interference between transverse bow and stern waves is important, and corresponds to the vast majority of ships. The predictions given by the Rabaud–Moisy 'cutoff-wavelength model' and the 'Gaussian pressure distribution model' are in close agreement with two wake observations for $1.6 < F < 1.7$ and may also be consistent with several wake observations for $0.6 < F < 1.4$, but are not consistent with most observations. This finding and a critical examination of the assumptions underlying the Rabaud–Moisy model and the Gaussian pressure distribution model suggest that these theoretical models may not be realistic for most ships. This conclusion is further validated by numerical computations of wave patterns for $F = 1$. The computed waves are largest along a ray angle that agrees with the prediction of the bow and stern waves interference model, but is noticeably smaller than predicted by the Gaussian pressure distribution model.

1. Introduction

The far-field waves generated by a ship hull, of length L_s, that advances at constant speed V_s along a straight path in calm water of large depth are considered. Main features of far-field ship waves, commonly called the Kelvin wake, have been explained by Kelvin and are well known. A main result of Kelvin's classical far-field analysis is that ship waves cannot exist outside a wedge, with half angle

$$\psi_K \approx 19°28' \quad (1)$$

from a ship track, that trails a ship. This angle is independent of the hull shape or the Froude number

$$F \equiv V_s/\sqrt{gL_s} \quad (2)$$

where g denotes the acceleration of gravity. However, numerous observations of ship wakes that are significantly narrower than the wake angle ψ_K expected from Kelvin's analysis have long been observed: e.g. [1–5]. This experimental fact is clear from the observations reported by Rabaud and Moisy in [5]. These

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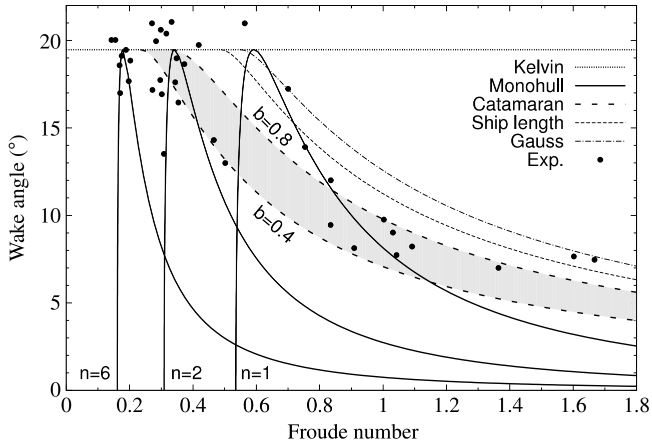

Fig. 1. Observations (Exp.) of ship wake angles ψ_{max} reported by Rabaud and Moisy in [5] and predictions given by three simple theoretical approaches. Specifically, the figure depicts the angles ψ_{max} predicted by the 'Gaussian pressure distribution model' (Gauss), the Rabaud–Moisy 'cutoff wavelength' model with the cutoff wavelength λ^{cut} taken as the ship length (Ship length), and lateral interference between the divergent waves created by the twin hulls of a catamaran (Catamaran) with beam/length ratio b within the range $0.4 \leq b \leq 0.8$. The figure also shows the ray angles ψ_n with $n = 1$, 2 and 6 (the curves that correspond to $n = 3$, 4, 5 are not shown because they are closely packed between the $n = 2$ and $n = 6$ curves) along which the waves are largest due to constructive longitudinal interference between the transverse and divergent waves created by the bow and the stern of a monohull ship (Monohull).

observations, reproduced here in Fig. 1, are within the range $13^\circ \leq \psi_{max} \leq 21^\circ$ and mostly located around the Kelvin angle ψ_K for Froude numbers $F < 0.6$, but are consistently and significantly smaller than ψ_K for $0.6 < F$.

Several alternative theoretical explanations of these wake observations in apparent variance with Kelvin's classical result have been proposed: e.g. [4–8]. The explanations offered in [5,8,9] are based on a linear potential-flow analysis of steady ship waves, unlike the considerably more complex explanations proposed in [2,4,6,7] that invoke effects of ambient waves, nonlinearities or finite water depth. These more complex theoretical explanations are not considered here; i.e. only the three alternative 'simple theories' recently offered in [5,8,9] are compared to one another and confronted to the 37 wake observations reported in [5]. These three theories have in common the fact that they each rely on a key assumption that greatly simplifies the analysis of the Kelvin wake of a ship, but differ in that the key assumptions underlying the alternative theories offered in [5,8,9] are markedly different.

The explanation proposed in [5] relies on the assumption that waves with wavelengths Λ greater than the ship length L_s can be ignored, i.e. on the restriction $\lambda \equiv \Lambda/L_s \leq 1$. This 'cutoff-wavelength model' is also justified in [5] via the 'Gaussian pressure distribution model', which is based on approximating the flow created by a ship hull by means of a Gaussian distribution of pressure at the free surface. This 'Gaussian pressure distribution model' is further considered in [8,10]. Although based on different assumptions, the Rabaud–Moisy cutoff-wavelength model and the Gaussian pressure distribution model yield similar predictions that the wake angle ψ_{max} of a ship decreases like $1/F$ as the Froude number F increases beyond $F \approx 0.5$.

The theoretical explanation proposed in [9] relies on the assumption that the wave pattern of a ship mostly consists of dominant waves that are created by the ship bow and stern, and is mostly determined by interference effects between these dominant waves. Specifically, the explanation offered in [9] relies on an elementary 'geometrical' analysis of interference between the dominant waves created by the bow and the stern of a monohull ship (longitudinal interference) or by the bows of the twin hulls of a catamaran (lateral interference). The 'dominant waves interference model' considered in [9] does not involve the amplitudes of the bow and stern waves, and is particularly simple.

Although comparisons with the observations of ship wakes reported in [5] are given in [9], this previous study does not include comparisons of the basic assumptions underlying the three alternative flow models considered in [5,8,9], i.e. the 'cutoff-wavelength model' offered in [5], the 'Gaussian pressure distribution model' considered in [5,8], and the 'dominant waves interference model' proposed in [9]. Moreover, no comparison of the theoretical predictions given by the alternative theories proposed in [5,8,9] is included in these studies. It is then interesting and useful – indeed necessary – to compare the assumptions underlying the three theoretical models proposed in [5,8,9] and to compare the theoretical predictions given by these alternative explanations to the 37 observations of ship wakes reported in [5] for the broad range of Froude numbers $0.1 < F < 1.7$. This comparison of underlying assumptions and predictions, not previously considered as already noted, is then considered here.

As already noted, the theoretical explanation offered in [5] is based on the assumption that waves with wavelengths Λ larger than the ship length L_s can be ignored. This key assumption is justified in [5] via the consideration of the waves created by a Gaussian distribution of pressure at the free surface, also considered in [8] as noted earlier. A Gaussian distribution of pressure at the free surface may be a reasonable model of the flow due to a high-speed planing hull. However, a free-surface pressure distribution that has a single peak may not realistically account for the strong interference between the dominant waves created by the bow and the stern of a ship, and therefore may not be a realistic model of the flow around the hull of a typical displacement ship (notably, monohull ships or catamarans). Moreover, the Gaussian distribution of pressure considered in [5,8] is smooth (infinitely differentiable), whereas the flow created by the bow and the stern of a ship (especially a fine bow or stern) varies very rapidly and indeed is not smooth at a sharp bow and stern, where the hull geometry varies abruptly. The difference is mathematically important because far-field waves are strongly influenced by near-field singularities, as shown in e.g. [11]. Indeed, this basic property of Fourier transforms provides a mathematical interpretation (explanation) of the common observation that a ship mostly creates two dominant waves that originate from points where the hull geometry varies abruptly, i.e. the ship bow and stern. This well-known feature of the wave-making of a ship – the key approximation underlying the dominant waves interference model considered in [9] – is clearly apparent from Fig. 2, which depicts the wave patterns created by the Wigley hull and the Series 60 model at a Froude number $F = 0.3$. These wave patterns were determined via the Neumann–Michell theory given in [12,13].

Thus, the smooth (Gaussian) free-surface pressure-distribution, with a single peak, considered in [5,8] may not be a realistic model of the flow around a ship hull, except for high-speed planing hulls, and consequently may not provide an adequate justification of the key assumption that underlies the 'cutoff wavelength' model of Kelvin's wake proposed by Rabaud and Moisy in [5]. It is then necessary to test if the assumption $\lambda \equiv \Lambda/L_s \leq 1$ that underlies the Rabaud–Moisy flow-model can be rationalized, notably in the light of the wave-interference analysis given in [9]. Indeed, it may be argued that the analysis of interference between the dominant waves created by the bow and the stern of a ship given in [9] provides a reasonable definition of a cutoff wavelength λ^{cut}; specifically, this definition is related to the fact that interference between dominant divergent waves results in the apparent effective elimination of wavelengths $\lambda^{cut} < \lambda$ via destructive interference.