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Initial wave breaking dynamics of Peregrine-type rogue waves: A numerical and experimental study

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ABSTRACT

The Peregrine breather, today widely regarded as a prototype for spatio-temporally localized rogue waves on the ocean caused by nonlinear focusing, is analyzed by direct numerical simulations based on two-phase Navier–Stokes equations. A finite-volume approach with a volume of fluid method is applied to study the Peregrine breather dynamics up to the initial stages of wave breaking. The comparison of the numerical results with laboratory experiments to validate the numerical approach shows very good agreement and suggests that the chosen method is an effective tool to study modulation instability and breather dynamics in water waves with high accuracy even up to the onset of wave breaking. The numerical results also indicate some previously unnoticed characteristics of the flow fields below the water surface of breathers, which might be of significance for short-term prediction of rogue waves. Recurrent wave breaking is also observed.

1. Introduction

The formation of rogue waves in the oceans is at present intensely debated to be related to modulation instability (MI) [1]. This instability was originally discovered in the context of Stokes waves and is thus also often referred to as Benjamin–Feir instability [2–5]. Today, it can be discussed most generically within the context of the nonlinear Schrödinger equation (NLS) [6,5], which is the lowest order model for weakly nonlinear dispersive wave envelope dynamics. The NLS is integrable [7] and has successfully proven to provide suitable initial and boundary conditions to allow observation of soliton dynamics in dispersive and nonlinear media. A special class of exact solutions of the NLS are the so-called breathers on finite background [8], which describe strong nonlinear focusing of waves, and therefore rogue wave dynamics. Among the different kinds of breather solutions, there is the Peregrine breather [9], which is localized in both time and space. Basically, it describes the nonlinear stage of the MI of Stokes waves for infinite modulation wavelength. Today the Peregrine breather is thus often considered to be the most likely prototype for rogue waves [10,11]. The Peregrine breather itself and related NLS solutions are currently intensely analyzed and studied in several nonlinear dispersive media [12–15]. The recent observations in optics [16,17], in water waves [18–22] and in plasma [23] have demonstrated impressively the ability of the NLS to model nonlinear focusing and spatio-temporal localization of wave groups.

For water waves, a number of studies on breather type dynamics and nonlinear focusing have become available in recent years, and all of them show remarkable correspondence between NLS theory and experimental data for wave states with small steepness, i.e. truly weak nonlinearity. The behavior of steeper, or more nonlinear waves, and the behavior of breaking, is far less understood, however. Interestingly, most available studies focus on the surface elevation dynamics only, and it seems that hardly any attention has been paid to the sub-surface flow fields of breather type water waves. In this study, we thus report direct numerical simulations (DNS) of Peregrine breather dynamics on the basis of two-phase incompressible Navier–Stokes equations. A finite volume method (FVM) is applied for discretization and a volume of fluid technique (VOF) for capturing the interface dynamics between water and air. A special focus is put on the dynamic evolution up to initiation of wave breaking, and on the sub-surface flow fields. The study may have a significant impact in applications in offshore engineering.

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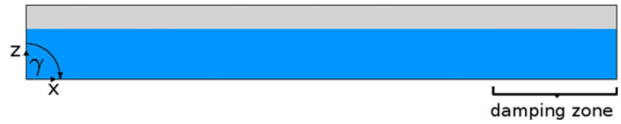

Fig. 1. The configuration of the solution domain. A grid deformation, generating the waves, initiates at $x = 0$ and a damping zone extends over 2.5 m on the opposite end.

2. Theoretical preliminaries and numerical setup

The temporal and spatial dynamics of deep-water wave packets can be described by the following form of the NLS [5,24],

$$-i\left(\frac{\partial A}{\partial t}+c_g\frac{\partial A}{\partial x}\right)+\frac{\omega_0}{8k_0^2}\frac{\partial^2 A}{\partial x^2}+\frac{\omega_0 k_0^2}{2}|A|^2A=0, \tag{1}$$

with the free surface elevation given by

$$\eta(x,t)=\mathrm{Re}\left(A(x,t)\exp\left[i\left(k_0x-\omega_0 t\right)\right]\right). \tag{2}$$

Here, k_0 and ω_0 denote the wave number and wave frequency, respectively. For deep-water conditions, the wave-packet $A(x,t)$ propagates with the group velocity $c_g = \frac{\omega_0}{2k_0}$ being half the phase speed of the waves. The NLS admits an infinite number of pulsating solutions on finite background, which describe the finite amplitude modulation instability dynamics of the Stokes solution $A_S(x,t) = a_0 \exp\left(-i\frac{a_0^2k_0^2\omega_0}{2}t\right)$ with an amplitude of a_0. The first family of breather solutions that was found are referred to as Akhmediev breathers [25,26]. The solutions are periodic in space and localized in time. The Peregrine breather [9] arises in the limit of infinite period of the Akhmediev breathers. This particular solution is localized in both space and time. It amplifies the amplitude of the background by a factor of three and is therefore considered to be an appropriate model to describe rogue wave dynamics in water waves as well as other nonlinear dispersive media, such as in nonlinear fiber optics [16] and in plasma [23]. The Peregrine solution of the NLS can be expressed as follows:

$$A_P(x,t)=a_0\exp\left(-i\frac{a_0^2k_0^2\omega_0}{2}t\right)\times\left(-1+\frac{4\left(1-ik_0^2a_0^2\omega_0 t\right)}{1+\left[2\sqrt{2}k_0^2a_0\left(x-c_gt\right)\right]^2+k_0^4a_0^4\omega_0^2t^2}\right). \tag{3}$$

It is the lowest-order solution of an infinite hierarchy of doubly-localized solutions, derived by nonlinear superposition of several Peregrine solutions [27]. The Peregrine breather dynamics has been recently confirmed in water wave experiments [18,19]. Subsequently, initial conditions for the numerical simulations as well as for the laboratory experiments are determined by evaluating Eq. (2) for a selected position x^*, see [28].

For our simulations we use a second-order finite volume discretization of the Navier–Stokes equations as implemented in the commercial STAR-CCM+ software [29]. The air and the water are assumed to have constant viscosity and density, with the density of air being $\rho_{\mathrm{air}} = 1.2$ kg m^3, the density of water being $\rho_{\mathrm{water}} = 1000$ kg m^3, the dynamic viscosity of air being $\mu_{\mathrm{air}} = 1.8\cdot10^{-5}$ Pa s and the dynamic viscosity of water being $\mu_{\mathrm{water}} = 0.001$ Pa s. For capturing the interface between air and water, a VOF method is used in this study [30].

The solution domain is based on the wave tank used in [19], see Fig. 1 for a schematic sketch.

The tank is initially filled up to a height of 1 m with water, while the rest of the tank volume is air at an initial reference pressure of $p_{\mathrm{ref}} = 1013.25$ hPa.

The origin of the reference coordinate system is located at the bottom left corner of the computational domain. The length in x-direction is $l_x = 12.0$ m and in z-direction is $l_z = 1.5$ m. The left wall can rotate around the y-axis, characterized by the angle γ relative to the x-axis. Thus, the wall moves like a flap type wave maker and can be used to generate waves just as in the experiments [18]. The tank sides as well as the tank bottom are modeled as no-slip walls. Towards the end of the experimental tank there is an absorbing beach. Analogously a numerical wave damping zone was introduced into the numerical model, extending over a length of 2.5 m next to the tank boundary opposite to the moving flap. The damping is implemented by applying a resistance to vertical motions [31]. For the vertical velocity component w the damping is achieved by adding a source term to the equations of motion, which has the following form:

$$q_z^{\mathrm{d}}=\rho(f_1+f_2|w|)\frac{\exp(\kappa)-1}{\exp(1)-1}w,\qquad \kappa=\left(\frac{x-x_{\mathrm{start}}}{x_{\mathrm{end}}-x_{\mathrm{start}}}\right)^{n_j}. \tag{4}$$

Here, x denotes the wave propagation direction with x_{start} being the start- and x_{end} the end-coordinate of the damping zone, respectively. For the present study, the model damping parameter values have been set to $f_1 = 10.0$, $f_2 = 10.0$ and $n_j = 2.0$. From the subsequent results it will be seen that this implementation functions very well. Nevertheless, both in experiments and in numerical analysis we have made sure that there are no spurious effects of reflections coming into play for the results that we discuss below.

The simulations describe the propagation dynamics of the breather in one spatial dimension. The solution domain was discretized with a rectilinear grid. The grid has 200 cells per carrier wavelength λ_0 and 16 cells per carrier wave amplitude a_0. It is gradually coarsened with increasing distance from the water surface, and the overall number of cells could be substantially reduced in comparison to a grid with constant cell volume, while yielding a comparable discretization error. A number of calculations have been performed to ensure satisfactory convergence of the discretization, and finally a mesh with 685, 325 cells has been selected for the results presented.

To generate waves, experimentally [18] it has turned out effective to harmonically move the flap with an amplitude proportional to the desired temporal surface elevation $\eta(x^*, t)$ of the water, which in turn is given from the NLS solution $A(x,t)$. To avoid causing disturbances by starting the flap movement with full amplitude, a linear fade-in for the first 4 s is applied. To realize the flap motion in the computational domain, we used moving control volumes, i.e. moving computational grid cells: the grid was deformed such that the movement of the left domain boundary corresponds to the flap movement. For that purpose, the STAR-CCM+ morpher tool was used. The morpher stretches or shrinks all grid cells in x-direction proportional to the distance between the flap wall and the fixed opposite wall, see Fig. 2.

The advantage of this approach is, that the number of grid cells remains constant. Moreover, the grid retains its quality throughout the morphing process since the flap movement, and thus also the resulting grid deformation, is rather small.

The calculations were carried out as a direct numerical simulation (DNS), i.e. no modeling was applied to the Navier–Stokes equations. Apart from the slight breaking of the rogue wave, the flow is mostly laminar. Thus, the temporal and spatial fluctuations can be resolved with an acceptable computational effort.

The STAR-CCM+ Implicit Unsteady solver was applied to carry out the transient computations. To ensure numerical stability in time-marching, a good guideline is that a fluid particle should not cross more than half a computational cell per time step, i.e. with the Courant number C, the stability condition is $C = u_i\Delta t/\Delta x_i < 0.5$, where u_i is the velocity component in x_i-direction, Δx_i labels the minimum cell size in x_i-direction and Δt denotes the time step