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A novel hybrid model for the motion of sustained axisymmetric gravity currents and intrusions

M. Ungarish [a], C.G. Johnson [b,*], A.J. Hogg [b]

[a] Department of Computer Science, Technion, Haifa 32000, Israel

[b] Centre for Environmental & Geophysical Flows, School of Mathematics, University of Bristol, UK

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ABSTRACT

We consider the sustained propagation of axisymmetric intrusions and gravity currents through linearly stratified or unstratified ambient fluids. Such flow configurations are found in a number of atmospheric and oceanic flows, in particular the predominantly horizontal spreading of a volcanic ash cloud after it has ascended through the atmosphere. There is strong theoretical evidence that these flows consist of two domains: an outer annular 'head' at the front of the current in which the motion is unsteady; and an inner, much thinner 'tail', which is steady, but spatially varying. The transition between the regions is a moving hydraulic jump. While it is possible to investigate these motions by numerically integrating the governing shallow layer equations, here we develop a much simpler mathematical model, which reproduces the more complicated model accurately and addresses issues such as what determines the position of the front and the moving bore between the two regions; what is the partition of influxed volume between the tail and head; and what is the distribution of suspended particles in the flow if present at the source? In such settings a conventional integral model fails, as does scaling based on dimensional analysis and the anticipation of an underlying self-similar form; the predictions they yield for these flows are incorrect. Instead we present a new hybrid model, which combines exact results of the steady shallow-water equations in the tail with simplifying assumptions in the head. This model predicts the flow properties by the straightforward solution of three ordinary differential equations (for front and bore positions and the volume fraction of particles in the head), without using adjustable constants, and obtains the correct asymptotic behaviour for the radius of the current r_N with respect to time t, namely $r_N \sim t^{4/5}$ for gravity currents and $r_N \sim t^{3/4}$ for intrusions. The predictions are obtained with negligible computational effort and accurately capture results from the more complete shallow water models. The model is also applied with success to gravity currents and intrusions that carry particles. For flows in which it is the presence of the particles alone that drives the motion, we identify length and time scales for the runout in terms of dimensional parameters that characterise the release, thus establishing the hybrid model as a useful tool also for modelling radial runout.

1. Introduction

In this contribution we study the sustained propagation (from a constant source) of axisymmetric intrusions and gravity currents at high Reynolds number. More specifically, we study the motion of a sustained volume flux of fluid flowing from a point source into a quiescent ambient, which is either unstratified or linearly stratified. If the ambient is stratified, this influx may form an intrusion about its level of neutral buoyancy (the height at which the density of the environment matches the density of influx). If instead the inflow is denser (or less dense) than any part of the ambient, there is no neutral buoyancy level, and the influx will flow as a gravity current over the horizontal base (or uppermost surface) of the ambient. Sustained, axisymmetric buoyancy-driven flows in both these regimes are observed in a range of environmental flows, including river outfalls [1], intrusions into stratified lakes [2] and volcanic plumes [3,4].

The buoyancy-driven spreading of intrusions is of particular practical importance due to the transport of volcanic ash by such flows. A volcanic plume rises from the vent until it reaches a height at which its density matches that of the atmosphere, whereupon it

* Corresponding author. Tel.: +44 7792640138.
E-mail addresses: unga@cs.technion.ac.il (M. Ungarish), chris.johnson@bristol.ac.uk (C.G. Johnson), a.j.hogg@bristol.ac.uk (A.J. Hogg).

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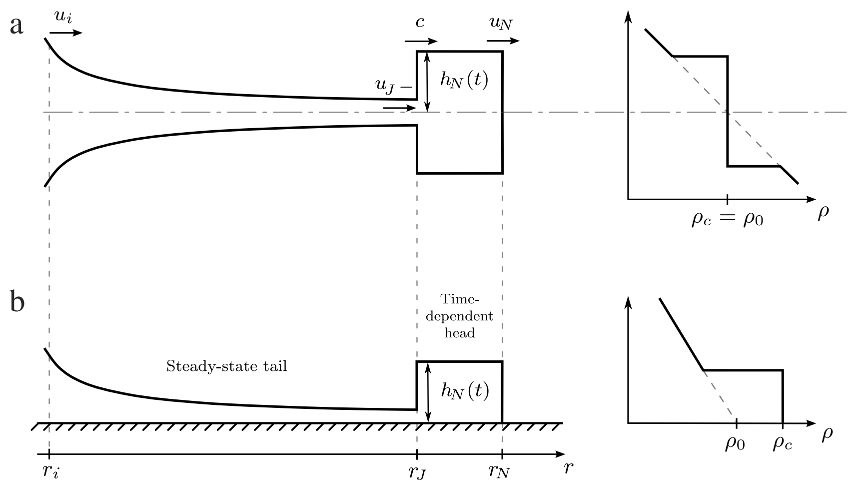

Fig. 1. Sketch of the radial height profile (left) and vertical density profile (right) of an intrusion (a), and a gravity current (b). In an intrusion, the buoyancy frequency, which measures the ambient density gradient, satisfies $\mathcal{N}^2 > 0$, the density of the current $\rho_c = \rho_0$ and $h(r, t)$ represents the half-height of the flow. In a gravity current, $\mathcal{N}^2 \geq 0$, $\rho_c \neq \rho_0$ (here, for currents denser than the ambient, $\rho_c > \rho_0$) and h represents the full height of the flow. Then for hybrid models, we assume that within the frontal region – the annulus between a hydraulic jump at $r = r_J(t)$ and the flow front at $r = r_N(t)$ – the flow height is spatially constant and equal to $h_N(t)$.

begins to spread horizontally. Importantly, the volumetric concentration of ash is sufficiently small so that it contributes only negligibly to the density of the volcanic cloud (see, for example, the typical volume fraction of particles at the top of the plume computed by Woodhouse et al. [5]). The intrusion spreads horizontally, therefore, because it has perturbed the background stratification, generating a well-mixed flowing layer (see Fig. 1). In the absence of wind, or if the wind speed is much less than the spreading rate of the intrusion, the ash cloud spreads radially, potentially transporting ash particles over considerable distances [4]. The challenge of predicting the dynamics of these clouds is important due to the significant hazard to aircraft flight that volcanic ash poses [6].

While the examples given so far have featured flows in which the density of the flowing layer remains constant, 'particle-driven' flows often arise in environmental settings, such as oceanic turbidity currents (see, for example [7–9], and references therein). In these flows, the presence of relatively heavy suspended particles contributes significantly to the overall density. These particles progressively sediment out of the flow, diminishing the density difference between the current and the ambient and reducing the driving force. Quantitative models of such flows necessarily couple the evolution of the suspension to the height and velocity of the flowing layer [7].

1.1. Shallow-layer models

One approach to modelling both gravity currents and intrusions exploits the thinness of the flows relative to their radial extent. In such thin flows, the excess pressure is hydrostatic to leading order and the flow is predominantly horizontal [10–12]. Both intrusions and gravity currents can be modelled within the same framework by including both the difference in density between the current and ambient and gradients of the ambient fluid density [12,13], and we present the equations in this form before considering the two types of flow separately. We denote the density of the intrusion or gravity current by ρ_c and the gradient of the ambient fluid density by $-\mathcal{N}^2\rho_c/g$, where $\mathcal{N}$ is the constant buoyancy frequency of the stably stratified ambient fluid. The reference density ρ_0 is the density of the ambient at the horizontal plane of symmetry of an intrusion, or at the base of a gravity current (Fig. 1). For a gravity current, $\rho_c > \rho_0$, and either $\mathcal{N} = 0$ (a uniform ambient) or $\mathcal{N} > 0$ (a stably stratified ambient). For an intrusion into a stratified ambient fluid centred about its neutral buoyancy height, $\rho_c = \rho_0$ and $\mathcal{N} > 0$. We note that although the density of an intrusion is the same as the average density of the fluid it displaces, it may nevertheless be thought of as a buoyancy-driven flow because the thickness of the intrusion, over which the density is uniform, means that there are density differences between the intruding fluid and the ambient.

The layer-averaged radial velocity of the flow is denoted as u and the flow thickness h, both functions of the radial coordinate, r, and time, t. While for a gravity current, h denotes the full thickness of the current, for an intrusion h denotes the half-thickness, and is measured from the neutral-buoyancy level to the upper interface; the lower interface is the mirror image (see Fig. 1).

Shallow-layer equations expressing the conservation of mass and balance of momentum have been developed to model the evolution of u and h, and are given by (see, for example [12,14, sections 13 and 16.3]),

$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rhu) = 0 \tag{1}$$

and

$$\frac{\partial}{\partial t}(uh) + \frac{1}{r}\frac{\partial}{\partial r}\left(ru^2h\right) + \frac{\partial}{\partial r}\left(\frac{1}{2}g'h^2 + \frac{1}{3}\mathcal{N}^2h^3\right) = 0 \tag{2}$$

where g' is the reduced gravity $(\rho_c - \rho_0)g/\rho_0$. Conservation of mass (1) is derived on the neglect of mixing with the surrounding fluid, while the momentum balance (2) is derived under the assumptions that drag is negligible, and that density differences are sufficiently small so that the flow is Boussinesq.

Gravity currents and intrusions may transport relatively dense particles in suspension, which settle with velocity w_s. Denoting the volume fraction of particulate by ϕ, a shallow layer model for its evolution is given by

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial r} = -\frac{w_s\phi}{d}, \tag{3}$$

where d denotes the total depth of the flowing layer; for gravity currents $d = h$, while for intrusions $d = 2h$. In this model it has been assumed that the flow is sufficiently turbulent to maintain a well-mixed suspension and that the suspension is sufficiently dilute that particle–particle interactions are prevented (see, for example [7]). The bulk density of the current is given by

$$\rho_c = \rho_f + (\rho_p - \rho_f)\phi, \tag{4}$$

where ρ_f and ρ_p are respectively the densities of the interstitial fluid and suspended particles. If $(\rho_p - \rho_f)\phi \ll |\rho_0 - \rho_f| + \rho_0$