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Numerical simulation of pressure-driven displacement of a viscoplastic material by a Newtonian fluid using the lattice Boltzmann method

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HIGHLIGHTS

- The pressure-driven displacement flow of a non-Newtonian fluid by a Newtonian fluid is studied.
- A two-phase lattice Boltzmann method is used.
- The various regularized viscoplastic models have been tested.
- Increasing the Bingham number and the flow index decreases the interfacial instabilities.

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ABSTRACT

The pressure-driven displacement of a non-Newtonian fluid by a Newtonian fluid in a two-dimensional channel is investigated via a multiphase lattice Boltzmann method using a non-ideal gas equation of state well-suited for two incompressible fluids. The code has been validated by comparing the results obtained using different regularized models, proposed in the literature, to model the viscoplasticity of the displaced material. Then, the effects of the Bingham number, which characterizes the behaviour of the yield-stress of the fluid and the flow index, which reflects the shear-thinning/thickening tendency of the fluid, are studied. It was found that increasing the Bingham number and increasing the flow index increases the size of the unyielded region of the fluid in the downstream portion of the channel and increases the thickness of the residual layer of the fluid resident initially in the channel; the latter is left behind on the channel walls by the propagating 'finger' of the displacing fluid. This, in turn, reduces the growth rate of interfacial instabilities and the speed of finger propagation.

1. Introduction

Pressure-driven displacement flows of one fluid by another having different fluid properties are common in many industrial processes, such as enhanced oil recovery [1], the transportation of crude oil in pipelines [2], fixed bed regeneration, hydrology and filtration. In food processing industries, cleaning also involves the removal of highly viscous material from conduits via displacement by water streams. In flow through porous media or in Hele-Shaw cells, the displacement of a highly viscous fluid by a less viscous one is accompanied by viscous fingering [3]. Achieving fundamental understanding of these flows became an active research area for decades [4].

The dynamics of displacement flows have been investigated both numerically and experimentally by several authors by considering miscible [5–12] as well as immiscible fluids [13–18]. It is well known that the displacement flow is always stable when the invading fluid is more viscous than the resident fluid [2]. When the displacing fluid is less viscous, the flow becomes unstable and "roll-up" (in miscible flows [1,19]) and sawtooth structures (in immiscible flows, [18]) appear at the interface separating the fluids. The linear instability in the three-layer/core-annular flow, which can be obtained when the elongated "finger" of the less viscous fluid penetrates into the bulk of the more viscous one, was also studied in immiscible [20–22] and miscible [19,23–26] systems.

In a Hele-Shaw cell, Goyal and Meiburg [7] studied numerically the miscible displacement flow of a highly viscous fluid by

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a less viscous one. They observed that the two-dimensional instability patterns become three-dimensional at higher flow rates. The flow field obtained in their simulation was qualitatively similar to that observed in the experiment of Petitjeans and Maxworthy [8] and the theoretical predictions of Lajeunesse et al. [27]. In the context of enhanced-oil recovery, Taghavi et al. [10,11] studied analytically and experimentally the displacement flow of two miscible fluids and observed Kelvin–Helmholtz like instabilities at low imposed velocities in the exchange flow dominated regime. Sahu et al. [9] investigated the effects of Reynolds number, Schmidt number, Froude number and angle of inclination in the pressure-driven flow of two miscible liquids of different densities and viscosities in an inclined channel. The behaviour of an infinitesimally small disturbance in such flows was also investigated by Sahu et al. [19] via a linear stability analysis.

The work discussed above considered only Newtonian fluids. In literature, to the best of our knowledge, very few studies has been carried out which investigated the displacement flow of viscoplastic materials. Below, we briefly review the previous work which studied the displacement flow of a non-Newtonian fluid by another Newtonian/non-Newtonian fluid.

Dimakopoulos and Tsamopoulos [28] studied the displacement of a viscoplastic material by air in straight and suddenly constricted tubes. They have shown that unyielded material arises in front of the air bubble and in the case of a constricted tube, near the recirculation corner. Papaioannou et al. [29], on the other hand, have studied the displacement of air by a viscoplastic fluid and revealed the conditions for the detachment of the viscoplastic material from the solid wall. Allouche et al. [30] and Wielage-Burchard et al. [31] studied the displacement flow of Bingham fluid by another fluid of same density in a plane channel. As the finger penetrates inside the channel a static residual layer of the displaced fluid is left behind the finger. They investigated the thickness of this residual layer for different Bingham numbers and compared their results with those obtained using the lubrication approximation.

The use of the discontinuous Bingham model for modelling the viscoplastic behaviour is not trivial because the yield surface is not known *a priori* but must be determined as part of the solution. Generally, viscosity regularization methods can be used with caution in order to overcome this difficulty. Frigaard and Nouar [32] studied the effects of different viscosity regularization models, such as the simple model [30], the Bercovier and Engleman model [33] and the Papanastasiou model [34] on the flow dynamics and found that the latter model performs better than the other two models. However, Frigaard and Nouar [32] remarked that the regularization methods should be used carefully in flow configurations, such as thin-film flows, by choosing very small values of the regularizing parameter.

Most of the numerical studies concerning displacement flows in the above review are for miscible systems, but few computational studies have been carried out on immiscible systems [14–17]. Numerical simulation of immiscible systems are expensive computationally due to the presence of sharp interfacial dynamics. During the past few decades, lattice Boltzmann method (LBM) has emerged as a promising technique for multiphase flow simulations [35]. The LBM is a mesoscopic model of fluid flows, which has its origins in kinetic gas theory. In the LBM, components of velocity and density are calculated by taking the moments of the distribution functions. It is a simple and elegant method having several other advantages, such as being easy to implement, with no need to resolve the interface explicitly, and massive parallel efficiency. The LBM involves only three explicit steps: (i) collision, (ii) streaming, and (ii) calculation of variables. Based on the class of problem of interest, researchers have been using different LBM approaches for multiphase flows, mainly, the colour segregation method of Gunstensen et al. [36], method of Shan and Chen [37], the free energy approach of Swift et al. [38] and the method of He and co-workers [39–41]. Using the method of Shan and Chen [37], the displacement flow of two immiscible liquids have been studied by several researchers [14–17]. The Reynolds number considered in these studies are very low, thus they did not observe any interfacial instabilities. Recently, Redapangu et al. [18] investigated the displacement flow of two immiscible Newtonian liquids at moderate Reynolds number using the method of He et al. [39]. They investigated the effects of the Atwood number, viscosity ratio, and angle of inclination on the flow dynamics and observed sawtooth-type waves at the interface separating the liquids. Also the lattice Boltzmann method has been used for viscoplastic fluid flows (see for examples Vikhansky [42,43] and Derksen [44]).

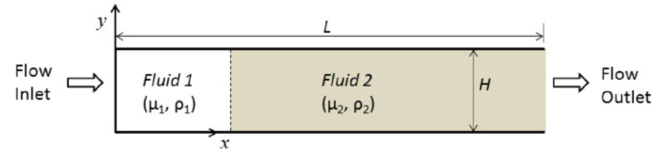

Fig. 1. Schematic showing the geometry (not to scale) and initial flow configuration. The inlet and outlet are located at $x = 0$ and $x = L$, respectively. The aspect ratio of the channel, L/H, is 48. Initially the channel is filled with fluids '1' and '2' from $0 \le x \le 5$ and $5 \le x \le L$ of the channel, respectively.

The buoyant displacement flow of one fluid by another fluid has been studied by several researchers (see [18] and references therein) and displacement flow of miscible viscoplastic fluids without density contrast has been studied by Frigaard and co-workers [30,31] as discussed above. Also as they were interested in investigating mud removal in the primary cementing of oil–gas well bore, they considered isodensity fluids in their studies. In the present work, the pressure-driven displacement flow of two immiscible liquids of different densities and viscosities is studied using a multiphase lattice Boltzmann method [39,45]. In order to achieve high computational efficiency, our LBM algorithm is implemented on a graphics processing unit (GPU) [46]. It is also important to note here that the work of Dimakopoulos and Tsamopoulos [28] and Papaioannou et al. [29] are restricted to air/viscoplastic material systems, whereas Wielage-Burchard et al. [31] is for density-matched materials. Our work provides a generalization of these studies and considers a different parameter range. Another important focus here is on the development of the LBM which is used to study the 2D problem first. This versatile, and massively-parallelisable method can then be extended readily to study the fully-3D problem, and to even include the effects of turbulence.

The rest of the paper is organized as follows. The details of the problem formulation and the LBM approach used to carry out the computations are provided in Section 2; the results are discussed in Section 3, and concluding remarks are given in Section 4.

2. Formulation

We consider the pressure-driven displacement of a viscoplastic, incompressible fluid of viscosity μ_2 and density ρ_2 (fluid '2') initially filled inside a horizontal two-dimensional channel. A Newtonian fluid (fluid '1') of viscosity μ_1 and density ρ_1 is injected from the inlet through an imposed pressure-gradient, as shown in Fig. 1. A rectangular coordinate system (x, y) is used to model the flow dynamics, where x and y denote the coordinates in the horizontal and the wall-normal directions, respectively. The channel inlet and outlet are located at $x = 0$ and L, respectively. The rigid and impermeable walls of the channel are located at $y = 0$ and H, respectively. The aspect ratio of the channel, L/H, is 48; we have considered sufficiently longer channel so that the Neumann boundary at the outlet of the channel is valid. g is the acceleration due to gravity acting in the negative y-direction.

2.1. Numerical method

The two-phase lattice Boltzmann method used in the present study is similar to that of He and co-workers [39–41]. Previously,