European Journal of Mechanics B/Fluids 49 (2015) 226–234

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Farfield waves created by a monohull ship in shallow water

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ARTICLE INFO

Article history:
Received 1 August 2014
Received in revised form
23 September 2014
Accepted 28 September 2014
Available online 16 October 2014

Keywords:
Ship wave signature
Interference effects
Shallow water
Bow waves

ABSTRACT

Interference effects between the dominant waves created by the bow and the stern of a monohull ship, of length L, that advances at constant speed V along a straight path in calm water of uniform finite depth D are considered. The largest waves due to constructive interference result in an apparent wake angle $\psi_{\max}$ that can differ greatly from the cusp or asymptote angles associated with the wave pattern of a ship when interference effects are ignored, as in Kelvin's classical analysis. Thus, wave interference has a very large effect on the wave signature of a ship in shallow water and cannot be ignored. Water-depth effects on the wake angle $\psi_{\max}$ are found to be insignificant for water depths $d^L \equiv D/L$ greater than $d^L_{\text{deep}} \approx 0.89$ or for Froude numbers $F \equiv V/\sqrt{gL}$ greater than $F_{\text{deep}} \approx 0.5/\sqrt{d^L}$. Furthermore, the water depth d^L has only a relatively small influence on the wake angle $\psi_{\max}$ for $1.5 < F$, and the deep-water approximation $\psi_{\max} \approx 0.14/F^2$ can then be used.

1. Introduction

The waves created by a ship, of length L, that advances at constant speed V along a straight path in calm water of uniform finite depth D are considered, as shown in Fig. 1. The Froude number F and the nondimensional water depths d^L and d^V are defined as

$$F \equiv \frac{V}{\sqrt{gL}} \equiv \sqrt{\frac{d^L}{d^V}}, \quad d^L \equiv \frac{D}{L}, \ d^V \equiv \frac{Dg}{V^2} \equiv \frac{d^L}{F^2} \tag{1}$$

where g denotes the acceleration of gravity. Finite water depth effects are only significant if $d^V < d^V_\infty$ with $d^V_\infty \approx 3$, and the water depth can then be regarded as effectively infinite for $d^V_\infty < d^V$. The flow due to the ship is observed from a Galilean frame of reference attached to the moving ship. We define the speed-scaled nondimensional coordinates and the corresponding Fourier variables (used further on)

$$\begin{aligned} &(x, y, z) \equiv (X, Y, Z)g/V^2 \quad \text{and} \\ &(\alpha, \beta, k \equiv \sqrt{\alpha^2+\beta^2}) \equiv (A, B, K)V^2/g. \end{aligned} \tag{2}$$

As shown in Fig. 1, the undisturbed free surface is chosen as the plane $z = 0$ with the z axis directed upward, and the x axis is taken along the ship path and directed toward the ship bow.

Main properties of the wave pattern created by a ship that steadily advances in calm water of uniform finite depth are well known, e.g. [1–4], and are now briefly summarized. The classical Kelvin deep-water wave pattern – determined via a farfield asymptotic analysis, based on the method of stationary phase, of the waves created by a 'ship' that is modeled as a flow disturbance concentrated at a point [5] – is greatly modified in finite water depth, as illustrated in Fig. 2. Specifically, this figure depicts the Kelvin wake in the deep-water limit $d^V = \infty$ (top left corner) and five wave patterns for finite water depths d^V that correspond to $d^V = 1.5, 1.1, 0.9, 0.5$ and 0.1.

The effect of finite water depth is most striking if $d^V < 1$. Indeed, the wave patterns (depicted in the top row of Fig. 2) for $1 < d^V$, i.e. in deep or relatively deep water, are qualitatively similar, although the cusp angle ψ_{cusp} and the wavelengths of the transverse and divergent waves greatly increase as the water depth d^V decreases from ∞ to 1. However, the wave patterns in the deep-water regime $1 < d^V$ and the shallow-water regime $d^V < 1$ depicted in the top and bottom rows of Fig. 2 are qualitatively different. Indeed, the (shallow-water) wave patterns for $d^V < 1$ (bottom row) have no transverse waves and no cusp line, but have an asymptote instead.

The angles of the asymptote (if $d^V < 1$) or the cusp (if $1 < d^V$) are denoted ψ_{asymp} or ψ_{cusp} hereafter, as in Fig. 3. This figure shows that the cusp angle ψ_{cusp} and the asymptote angle ψ_{asymp} are both equal to 90° for $d^V = 1$, and vary rapidly for values of the water depth d^V in the vicinity of the critical water depth $d^V = 1$, where

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http://dx.doi.org/10.1016/j.euromechflu.2014.09.006
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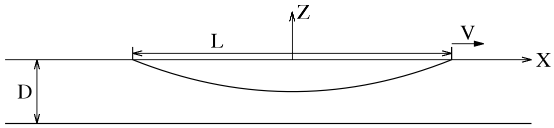

Fig. 1. Side view of a monohull ship, of length L, that advances at constant speed V along a straight path in calm water of uniform finite depth D.

the transition between the deep-water and shallow-water wave patterns depicted in the top and bottom rows of Fig. 2 occurs. Fig. 3 also shows that the cusp angle ψ_{cusp} does not differ significantly from the deep-water Kelvin cusp angle $\psi_{\text{Kelvin}} \approx 19°28'$ for $2 < d^V$.

As already noted, the farfield wave patterns depicted in Fig. 2 correspond to a nearfield flow disturbance concentrated at a point, in the manner shown by Kelvin [5]. This particularly crude approximation of the nearfield flow created by a ship, modeled as a Dirac pressure distribution $\delta(x)\delta(y)$ at the origin of the wave pattern, yields wave patterns that only depend on the coordinates $(x, y) \equiv (X, Y)g/V^2$ and the speed-scaled water depth $d^V \equiv Dg/V^2$, i.e. that do not depend on the ship length L or the Froude number F. In particular, the cusp angle ψ_{cusp} and the asymptote angle ψ_{asymp} of the wave patterns depicted in Fig. 2 are independent of F and only depend on d^V, as shown in Fig. 3.

The crude model of the nearfield flow created by a ship used by Kelvin turns out to be adequate for low Froude numbers $0 < F \leq F_K$, with $F_K \approx 0.6$ in deep water. However, it has long been observed that the waves created by a ship at high Froude numbers $F_K < F$ in deep water are mostly found within wedges that are significantly and consistently narrower than the Kelvin cusp angle $\psi_{\text{Kelvin}} \approx 19°28'$; [6–10]. Several theoretical explanations of these observations have been proposed in [9–17]. Within the context of linear inviscid waves considered by Kelvin as well as here and in [1–4,10,13–17], waves can neither grow nor be attenuated due

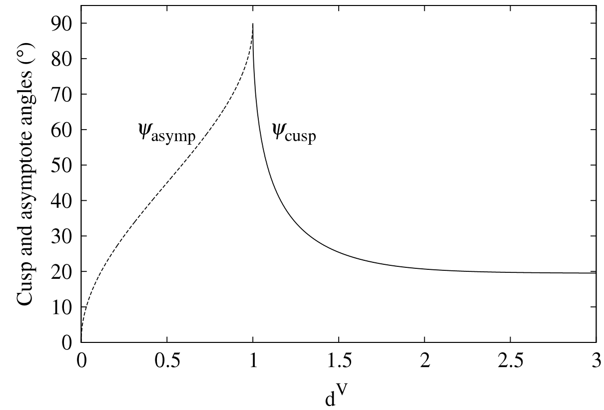

Fig. 3. Variations of the asymptote angle $\psi_{\text{asymp}}(d^V)$ for $0 \leq d^V \leq 1$ (dashed line) and of the cusp angle $\psi_{\text{cusp}}(d^V)$ for $1 \leq d^V$ (solid line) with respect to $0 \leq d^V \leq 3$.

to nonlinear or dissipative effects, and wave interference therefore is the only flow physics there is and the only flow physics that is needed to explain ship wave patterns.

Within the framework of the theory of linear inviscid waves, the flow around a ship hull can be represented by means of a distribution of sources and sinks over the ship hull surface, e.g. as specified by the Neumann–Michell theory [18,19] or the related Hogner approximation. Thus, *longitudinal* interference occurs between the waves created by the sources and the sinks distributed over the bow and stern (fore and aft) regions of a ship hull, and *lateral* interference occurs between the sources (or sinks) distributed over the port and starboard sides of the bow (or stern) regions of the hull. The farfield waves created by a ship are then the result of both longitudinal and lateral interference effects, and the wave signature of a ship depends on the length L of the ship, i.e. on

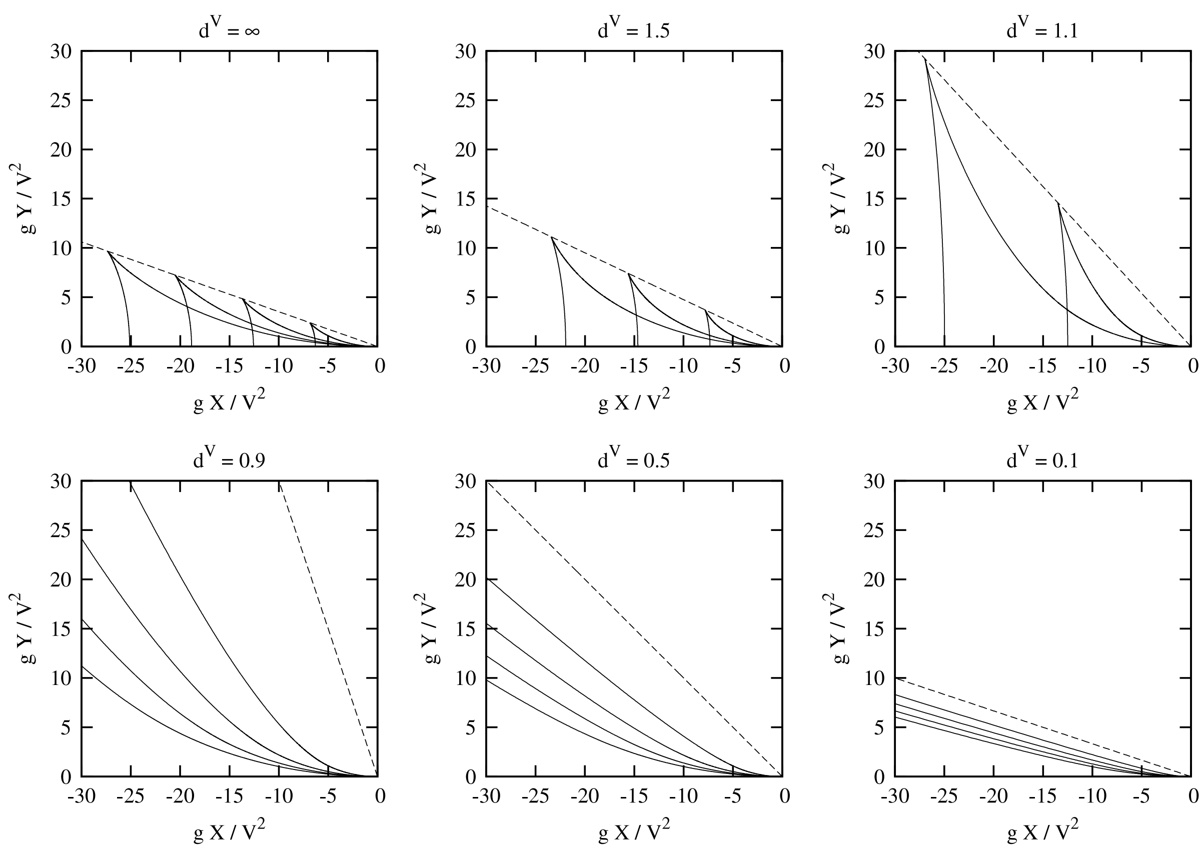

Fig. 2. Steady ship wave patterns in uniform finite water depth for $d^V = \infty$ (left column), 1.5 (center) and 1.1 (right) in the top row, and for $d^V = 0.9$ (left column), 0.5 (center) and 0.1 (right) in the bottom row. The cusp lines for $1 < d^V$ (top row) and the asymptote angles for $d^V < 1$ (bottom row) are marked by dashed lines.