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Evaluation of ship waves at the free surface and removal of short waves

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ABSTRACT

The dual basic tasks of evaluating ship waves at the free surface and of removing unwanted short waves are considered within the framework of the 'free-surface Green function potential flow theory', based on a Green function that satisfies the radiation condition and the Kelvin–Michell linearized boundary condition at the free surface. A practical approach based on parabolic extrapolation within an extrapolation layer bordering the free surface is used. The height of the extrapolation layer is defined explicitly via simple analytical relations in terms of the Froude number and the slenderness of the ship hull, and varies from the bow to the stern. The bow-to-stern variation is an important ingredient that accounts for the fact that waves along the ship hull aft of the bow wave differ from the bow wave. Indeed, a ship bow wave is significantly higher and shorter than waves aft of the bow wave, is affected by nearfield effects related to the rapid variation of the hull geometry at a ship bow, and consequently contains more short wave components. Illustrative calculations demonstrate the need for removing short ship waves and the effectiveness of the approach based on parabolic extrapolation.

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1. Introduction

An offshore structure (or any other floating body without mean forward speed) in ambient time-harmonic (regular) waves with frequency ω generates a system of (diffracted–radiated) waves that all have the same frequency ω and wavelength $\lambda = 2\pi g/\omega^2$. Such a system of waves, with a single *discrete* wavelength, differs in a fundamental respect from the waves – with a continuous spectrum of wavelengths – that are generated by a ship advancing in calm water or in ambient regular waves. In particular, for the simplest case of a ship advancing with forward speed V_s in calm water considered here, the ship creates waves with a spectrum of wavelengths λ within the range $0 \leq \lambda \leq 2\pi V_s^2/g$. Thus, ship waves defined within the classical framework of potential flow theory include very short waves that can be significantly affected by surface tension and viscosity, and consequently are physically unrealistic. The spectrum of ship waves may also include waves that are not appreciably affected by surface tension and viscosity but are short with respect to the ship length L_s, and consequently may have a limited effect on the ship drag, sinkage and trim. There is then a practical need for eliminating short waves in the spectrum of waves generated by ships. Indeed, an effective and practical method for filtering short gravity waves is an important ingredient of any numerical method for computing ship waves. This basic issue is considered here within the framework of the 'free-surface Green function potential flow theory', based on a Green function that satisfies the radiation condition and the Kelvin–Michell linearized boundary condition at the free surface.

Thus, we consider linear potential flow about a ship hull of length L_s that steadily advances at speed V_s along a straight path in calm water of effectively infinite depth and lateral extent. The flow about the ship hull is observed from a righthanded moving system of orthogonal coordinates $\mathbf{X} \equiv (X, Y, Z)$ attached to the ship, and thus appears steady with flow velocity given by the sum of an apparent uniform current $(-V_s, 0, 0)$ opposing the ship speed V_s and the (disturbance) flow velocity $\mathbf{U} \equiv (U, V, W)$ due to the ship. The X axis is chosen along the path of the ship and points toward the ship bow. The Z axis is vertical and points upward, with the mean (undisturbed) free surface taken as the plane $Z = 0$. The length L_s and the speed V_s of the ship are used to define nondimensional coordinates $\mathbf{x} \equiv \mathbf{X}/L_s$ and flow potential $\phi \equiv \Phi/(V_s L_s)$.

We define the usual Froude number $F \equiv V_s/\sqrt{gL_s}$ where g is the acceleration of gravity. We also define the Froude number $F_{BD} \equiv V_s/\sqrt{gL^{BD}}$ based on a transverse dimension L^{BD} of the ship hull that is chosen as $L^{BD} \equiv BD/(B/2 + D)$. We have $L^{BD} < B$ and $L^{BD} < 2D$. The Froude numbers F and F_{BD} based on the length L_s or the transverse dimension L^{BD} of the ship are related as

$$F_{BD} \equiv \frac{F}{\sqrt{\sigma^H}} \quad \text{where } \sigma^H \equiv \frac{L^{BD}}{L_s} \equiv \frac{bd}{b/2 + d}$$

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$$\text{with } b \equiv \frac{B}{L_s} \text{ and } d \equiv \frac{D}{L_s}. \tag{1}$$

For a beam/length ratio $b = 0.15$ and a draft/length ratio $d = 0.05$, the hull slenderness is $\sigma^H = 0.06$.

As already noted, the flow about the ship is considered within the framework of the 'free-surface Green function method' based on a Green function $G(\widetilde{\mathbf{x}}; \mathbf{x})$ that satisfies the radiation condition and the Kelvin–Michell linearized boundary condition at the free surface. The points $\mathbf{x} \equiv (x, y, z)$ and $\widetilde{\mathbf{x}} \equiv (\widetilde{x}, \widetilde{y}, \widetilde{z})$ stand for 'boundary points' located on the ship hull surface Σ^H, and 'flow-field points' located on Σ^H or in the flow region outside Σ^H.

Within this approach, the flow potential $\widetilde{\phi} \equiv \phi(\widetilde{\mathbf{x}})$ at a flow field point $\widetilde{\mathbf{x}}$ can be expressed as $\widetilde{\phi} = \widetilde{\phi}^L + \widetilde{\phi}^W$ where $\widetilde{\phi}^L$ represents a nonoscillatory local flow component and $\widetilde{\phi}^W$ defines the waves generated by the ship. These local and wave components are associated with the decomposition $G = L + W$ of the Green function G into a local flow component L and a wave component W, as in e.g. [1]. The local flow potential $\widetilde{\phi}^L$ is not considered here.

Thus, we only consider the wave potential $\widetilde{\phi}^W$. Within the free-surface Green function approach, the wave potential $\widetilde{\phi}^W \equiv \phi^W(\widetilde{\mathbf{x}})$ at a flow field point $\widetilde{\mathbf{x}} \equiv (\widetilde{x}, \widetilde{y}, \widetilde{z} \leq 0)$ is expressed as a Fourier superposition of elementary plane waves $\widetilde{E}$. Specifically, $\widetilde{\phi}^W$ is given by

$$\widetilde{\phi}^W = \frac{1}{\pi} \Im\mathrm{m} \int_{-\infty}^{\infty} dk \, \widetilde{S} \, \widetilde{E}$$

$$\text{with } \widetilde{E} \equiv e^{(1+k^2)\widetilde{z}/F^2 + i\sqrt{1+k^2}\,(\widetilde{x}+k\widetilde{y})/F^2} \tag{2}$$

e.g. [2,3]. The amplitude $\widetilde{S} \equiv \widetilde{S}(k; \widetilde{x})$ of the elementary waves $\widetilde{E}$ in the Fourier representation (2), called the wave-spectrum function or Kochin function, is also given by a Fourier superposition of elementary plane waves. Specifically, the wave-spectrum function $\widetilde{S}$ in (2) is given by a distribution of elementary waves E over the portion $\widetilde{\Sigma}^H$ of the mean wetted ship hull surface Σ^H that is defined by $\widetilde{x} \leq x$. We then have

$$\widetilde{S} \equiv \frac{1}{F^2} \int_{\widetilde{\Sigma}^H} da \, A \, E \quad \text{with } E \equiv e^{(1+k^2)z/F^2 - i\sqrt{1+k^2}\,(x+ky)/F^2}. \tag{3}$$

Here, $da \equiv da(\mathbf{x})$ is the differential element of area at a point $\mathbf{x}$ of the ship hull surface Σ^H.

An important particular case of the generic Fourier–Kochin representation (2)–(3) of ship waves is the wave potential $\widetilde{\phi}^W_H$ associated with the Hogner slender-ship potential $\widetilde{\phi}_H$ given in [4]. In this special case, the amplitude A of the elementary wave E in the wave-spectrum function $\widetilde{S}$ is given by

$$A = A^H \quad \text{with } A^H \equiv n^x. \tag{4}$$

Here, $\mathbf{n} \equiv (n^x, n^y, n^z)$ is a unit vector that is normal to Σ^H at $\mathbf{x}$ and points outside Σ^H, i.e. into the water. The Hogner slender-ship approximation (4), defined explicitly in terms of the hull geometry, is useful for many practical applications. Furthermore, this explicit flow approximation is a major element of the Neumann–Michell (NM) theory of ship waves given in [3] and considered further on. Indeed, the NM theory provides a correction of the Hogner slender-ship potential $\widetilde{\phi}_H$ given in [4].

The wavelength λ of the elementary waves $\widetilde{E}$ and E in (2) and (3) is given by

$$0 \leq \lambda \equiv 2\pi F^2/(1+k^2) \leq 2\pi F^2 \equiv \lambda_0 \tag{5a}$$

where λ_0 is the wavelength of the transverse waves generated by a ship along its track. Transverse and divergent ship waves correspond to $|k| < 1/\sqrt{2} \approx 0.71$ and $1/\sqrt{2} < |k|$, respectively; e.g. [5]. The ratio

$$\lambda/\lambda_0 = 1/(1+k^2) \tag{5b}$$

is equal to 1/2 for $k = 1$, 1/3 for $k = \sqrt{2}$, 1/5 for $k = 2$, 1/10 for $k = 3$, and approximately 6% or 4% for $k = 4$ or $k = 5$. Thus, values of $|k|$ greater than 3 correspond to waves that are significantly shorter than the longest waves created by a ship.

The continuous spectrum of waves generated by a ship can be usefully divided into 'long waves' associated with the range $-k_{\text{long}} \leq k \leq k_{\text{long}}$ where the cutoff wavenumber k_{long} can reasonably be taken as 3 or even 2, and 'short waves' that correspond to $k_{\text{long}} \leq |k|$. These short waves can be further divided into 'short gravity waves' that are too long to be significantly affected by surface tension and viscous effects, and 'very short waves' for which surface tension and viscous effects cannot be ignored. An elegant physics-based theory that accounts for the influence of surface tension and viscosity on gravity waves generated by a ship hull advancing in calm water or in ambient time-harmonic waves is expounded in [6–9]. Surface tension and viscosity are ignored here for simplicity, and because the cutoff wavenumber k_{long} is expected to correspond to relatively long waves not significantly affected by surface tension and viscosity (although that may not always be the case at model scale).

For a fully submerged body, for which $z \leq -\delta$ where $0 < \delta$ is the distance between the free surface plane $z = 0$ and the highest point of the submerged body surface (point nearest the mean free surface), we have $|E| \leq e^{-(1+k^2)\delta/F^2}$. Thus, the elementary wave function E and consequently the spectrum function $\widetilde{S}$ decay exponentially as $k \to \pm\infty$ for a fully submerged body. The Fourier integral (2) therefore converges for every value of $\widetilde{z} \leq 0$, and does not contain short waves, in this special case.

However, the situation is different, and more complicated, for a surface-piercing ship hull Σ^H, for which $z \leq 0$ and the function E does not decay exponentially as $k \to \pm\infty$. If $\widetilde{z} \leq -h < 0$, i.e. for flow field points $\widetilde{\mathbf{x}}$ at some distance below the mean free surface $\widetilde{z} = 0$, the exponential function $\widetilde{E}$ in (2) decays exponentially in the limit $k \to \pm\infty$ and the Fourier integral (2) can be evaluated accurately. In practice, the infinite limits of integration in (2) are replaced by finite limits $\pm k_\infty$. The function $\widetilde{E}$ is smaller than 0.7% if $(1+k^2)\widetilde{z}/F^2 < -5$, and (2) can be evaluated accurately for $\widetilde{z} \leq -h_\infty$ with

$$h_\infty \equiv 5F^2/(1+k_\infty^2). \tag{6}$$

Convergence of the Fourier integral (2) is not a priori obvious for $-h_\infty < \widetilde{z} \leq 0$, i.e. for flow field points $\widetilde{\mathbf{x}}$ in the vicinity of the free surface $\widetilde{z} = 0$. Indeed, we have $\widetilde{E} = 1$ if $\widetilde{z} = 0$, for every value of the Fourier variable k.

Furthermore, the relation (6) yields $k_\infty \to \infty$ as $h_\infty \to 0$, and therefore implies that the wave potential $\widetilde{\phi}^W$ includes very short waves in this limit. However, very short gravity waves are unrealistic because surface tension and viscous effects, ignored in (2) and (3), cannot be neglected in the short-wave limit $k \to \infty$, and because short waves that correspond to $k_{\text{long}} \leq |k|$ are of limited interest for most practical applications. Thus, robust evaluation of the wave integral (2) for $-h_\infty < \widetilde{z} \leq 0$ is a nontrivial basic issue for the computation of ship waves within the framework of the free-surface Green function approach.

A practical way of evaluating the Fourier integral (2) at and near the mean free surface $\widetilde{z} = 0$ is then required. To this end, the Fourier representation (2) can be modified as

$$\widetilde{\phi}^W = \frac{1}{\pi} \Im\mathrm{m} \int_{-k_\infty}^{k_\infty} dk \, \Lambda \, \widetilde{S} \, \widetilde{E}. \tag{7}$$

Selection of an appropriate finite limit of integration (cut-off wavenumber) k_∞, and an effective short-wave filter function Λ are important elements of the free-surface Green function theory of ship waves as already noted.