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2D numerical simulation of density currents using the SPH projection method

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ABSTRACT

Density currents (DCs) or gravity currents are driven by gravity in a fluid environment with density variation. Smoothed Particle Hydrodynamics (SPH) has been proved to have capabilities such as free surface modeling and accurate tracking of the immiscible-fluids interface that can be useful in the context of gravity currents. However, SPH applications to gravity currents have been limited to often-coarse simulations of high density-ratio currents. In this work, the SPH projection method is tried to solve currents with very low density-ratios (close to one), at a resolution, that captures the Kelvin–Helmholtz instabilities at the fluids interface. Existing implementations of the SPH projection method do not allow for an efficient solution of the currents with very low density-ratios. We found that a pressure-decoupling scheme inherently changes the way of distinguishing between the light and dense fluids in the projection method. Using this technique, SPH results, in a 2D lock-exchange flow at a Grashof number of 1.25×10^6 and Schmidt number of unity, are successfully compared with a previous grid-based simulation. Although these types of problems are well addressed in the past through grid-based methods, reproducing them with SPH reveals method's capabilities such as free surface modeling as advantages that can be benefited from in addressing relevant currents. DCs are often turbulent, but unfortunately, turbulence modeling is not yet well established in the SPH method. Nevertheless, there are opinions in the literature that SPH possesses some built-in mechanisms that compensate for the missing energy dissipation at the sub-particle scales. The results of our simulation and energy budget analysis of another lock-exchange flow at different levels of spatial resolution support this hypothesis, although do not provide conclusive evidence.

1. Introduction

Density currents (DCs) are originated under the influence of the gravity field in a fluid environment with variable density. The density-variation within the fluid environment can arise from a non-uniform concentration field of a solute material or from a non-uniform temperature field. In the present study, the SPH Lagrangian method is considered for accurate simulation of very low density-ratio (close to one) DCs.

SPH is a particle based Lagrangian method that was originally developed by Lucy [1] and separately and simultaneously by Gingold and Monaghan [2]. Initial applications of SPH were limited to the context of astrophysics and compressible flows. However, in the past decade, it has been largely applied to incompressible fluid mechanics problems. There are two general approaches to impose incompressibility in SPH. The more traditional one is the weakly compressible SPH (WCSPH) method, which was first proposed by Monaghan [3]. The second approach is the SPH projection method or truly incompressible SPH (ISPH), which has been developed more recently and was initially proposed by Cummins and Rudman [4]. The present work is based on ISPH. There are several types of ISPH algorithms available in the literature, mainly differing in their criteria for incompressibility. The original version, proposed by Cummins and Rudman [4], uses the divergence of the velocity field as a measure of deviation from incompressibility, i.e., as the source term of the Poisson equation. In another type of ISPH, Shao and Lo [5] have directly measured the density variation of each particle by the kernel summation over the neighboring particles. Hu and Adams [6] have proposed a combinational algorithm that imposes both the velocity-divergence-free and the zero-density-variation conditions by solving the Poisson equation twice in each time step. Xu et al. [7] have compared the three mentioned algorithms and concluded that by using the velocity-divergence as a source term, the method results in accurate solutions, but it would not be very stable. In comparison, by using the density-variation as a source term, method's stability improves, but it becomes less accurate. Finally, the combinational algorithm of Hu and Adams [6] is both accurate and stable, however, at the expense of being very time consuming. In order

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to benefit from the accuracy of the divergence-free method and still avoiding high computational cost, Xu et al. [7] have proposed a solution for the instability of this method, which shifts particles (in a direction that avoids particle clustering) at the end of each time step and then corrects their hydrodynamic properties using the Taylor series. This algorithm is used in the present study.

In recent years, the SPH method has been developed considerably for the simulation of multi-phase flows. For instance, one can refer to the publications of Hu and Adams [6,8,9] and Grenier et al. [10]. However, DCs such as salt-water flows in ambient fresh water have characteristics that separate them from multi-phase flows. A salt-water DC has only one phase, which contains a solute (salt) that has not been uniformly dissolved. In addition, the density ratio between the light and dense fluids is close to one, and they are miscible (density varies smoothly over the domain). Noticeably, these characteristics are in contrast to what is usually called a multi-phase flow. Using the SPH method, there are very few attempts to simulate DCs with the above-mentioned properties [11–13], which they have also been considered larger value of density ratio than that of saline to fresh water. Another shortcoming is that the Kelvin–Helmholtz instabilities at the interface of the dense layer and the lighter fluid have not been clearly captured in the simulations. In Section 3, it is discussed that decoupling of the pressure variable into two distinct terms facilitates simulation of the DCs with very small density differences. Similar but not the same pressure decoupling has been previously examined by Morris et al. [14] and Sigalotti et al. [15] in single-phase flows.

In this study, SPH is recruited as a direct numerical solver for two different test cases of 2D lock-exchange flows. In the first case, results are compared against the high-resolution grid-based DNS simulation of Hartel et al. [16] with a promising agreement. The second case, with a fourfold larger Grashof number, is studied to assess how the results of the SPH simulation of a 2D turbulent DC depend on the level of spatial resolution (i.e., number of particles).

Density currents usually occur in large scales and are consequently likely to be turbulent. Therefore, for a numerical tool dealing with such flows, it is essential to be armed with turbulence models. Indeed, the SPH method has some interesting features to be a candidate for DC simulation, regarding its Lagrangian nature and its well-known capability in free surface modeling. However, SPH has not been known to be a good framework for turbulence modeling. Several attempts have been made in Large Eddy Simulation [17], Reynolds Stress models [18], and a more recent SPH-ε method [19]; nonetheless, no well-established SPH turbulence model is available to date. Concurrently, there are some opinions in the literature that SPH has some built-in dissipation mechanisms that compensate for the missing energy dissipation at the sub-particle scales, or even more, it possesses a form of LES model hidden in its scheme [17,20]. Robinson and Monaghan [21] have recently re-cited these hypotheses and proposed that SPH can be considered as a direct numerical solver without turbulence modeling. Our numerical study on the 2D lock-exchange flow supports this hypothesis.

2. Governing equations

For the flows considered in this work, the continuity, momentum, and concentration equations take the following forms:

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P + \boldsymbol{g} + \nu\left(\nabla^2 \boldsymbol{u}\right), \tag{2}$$

$$\frac{DC}{Dt} = D\left(\nabla^2 C\right), \tag{3}$$

where $\boldsymbol{g}$, $\boldsymbol{u}$, and P stand for gravity acceleration, velocity, and pressure, respectively, and C is the concentration of the mixture, which will be defined in the next section. The physical properties of the mixture are the kinematic viscosity ν, the coefficient of the concentration diffusion D, and the density ρ. The time derivatives are written in the material or total form in accordance with the Lagrangian nature of the SPH method used here. Indeed, through SPH discretization, the continuum domain of the problem is replaced by discrete moving particles that carry flow variables, including velocity, pressure, and concentration in the current study.

3. Simulation of the density currents with close-to-one density ratios

3.1. Pressure decoupling and non-dimensionalization

In this section, the pressure decoupling is implemented, and the governing equations are non-dimensionalized. Then after introducing the projection algorithm in Section 3.2, the modified equations, obtained here, and their original form will be compared from the viewpoint of implementing the projection algorithm.

The pressure-decoupling scheme is

$$P = P' + \frac{1}{2}\left(\rho_0 + \rho_1\right) g\, y, \tag{4}$$

where ρ_0 and ρ_1 are densities of the light (e.g., fresh water) and dense (e.g., saline mixture at its maximum concentration) fluids, respectively, and y represents the vertical coordinate. Substitution of Eq. (4) into Eq. (2) results in

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P' + \frac{\left(\rho - \frac{1}{2}\left(\rho_0 + \rho_1\right)\right)}{\rho}\boldsymbol{g} + \nu\left(\nabla^2 \boldsymbol{u}\right). \tag{5}$$

The above equation is non-dimensionalized by defining the following parameters:

$$\boldsymbol{u}^* = \boldsymbol{u}/V, \qquad P^* = \frac{P'}{\rho_0 V^2}, \qquad t^* = \frac{t}{L/V}, \qquad \nabla^* = L\,\nabla, \tag{6}$$

where L is a characteristic length, and V is a characteristic velocity which we define as

$$V = \sqrt{\frac{\left(\rho_1 - \rho_0\right) gL}{\rho_0}}. \tag{7}$$

The coordinates x and y will be written in the dimensionless form as well, from here on, including the test cases. The non-dimensional form of Eq. (5) becomes

$$\frac{D\boldsymbol{u}^*}{Dt^*} = -\frac{\rho_0}{\rho}\nabla^* P^* + \frac{\rho_0}{\rho} C\,\boldsymbol{e}^g + \frac{1}{\sqrt{Gr}}\left(\nabla^{*2}\boldsymbol{u}^*\right), \tag{8}$$

where $\boldsymbol{e}^g$ is a unit vector in the direction of gravity, and Gr shows the Grashof number

$$Gr = \left(\frac{\sqrt{(\rho_1 - \rho_0)g/\rho_0}\,L^{3/2}}{\nu}\right)^2, \tag{9}$$

and finally, C, the concentration, has been defined as

$$C = \frac{\rho - \frac{1}{2}\left(\rho_0 + \rho_1\right)}{\rho_1 - \rho_0}. \tag{10}$$

By this definition, the concentrations of -0.5 and 0.5 are assigned for the light and dense fluids, respectively. From numerical point of view, this kind of pressure decoupling (Eq. (4)) is more beneficial in comparison to decoupling as $P = P' + \rho_0 gy$, which leads to the extreme concentrations of 0 and 1. Because particles accelerate in proportion to their concentration, former decoupling provides a balance between the light and dense particles acceleration. This is